CALCULUS BC
SECTION I, Part B
Time—50 minutes
Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

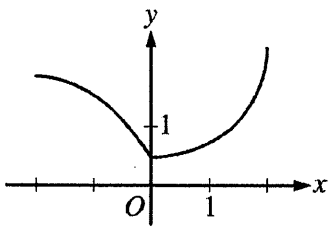
YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f

76. The function f, whose graph is shown above, is defined on the interval $-2 \le x \le 2$. Which of the following statements about f is false?

(A) f is continuous at $x = 0$.

(B) f is differentiable at $x = 0$. (sharp point at $x=0$)

(C) f has a critical point at $x = 0$.

(D) f has an absolute minimum at $x = 0$.

(E) The concavity of the graph of f changes at $x = 0$.

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77. Let f and g be the functions given by $f(x) = e^x$ and $g(x) = x^4$. On what intervals is the rate of change of $f(x)$ greater than the rate of change of $g(x)$?

$f'(x) = e^x \quad g'(x) = 4x^3$

(A) (0.831, 7.384) only

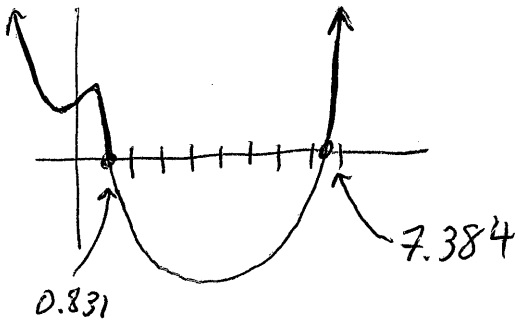
(B) $(-\infty, 0.831)$ and $(7.384, \infty)$

(C) $(-\infty, -0.816)$ and $(1.430, 8.613)$

(D) $(-0.816, 1.430)$ and $(8.613, \infty)$

(E) $(-\infty, \infty)$

Graph $y = e^x - 4x^3$



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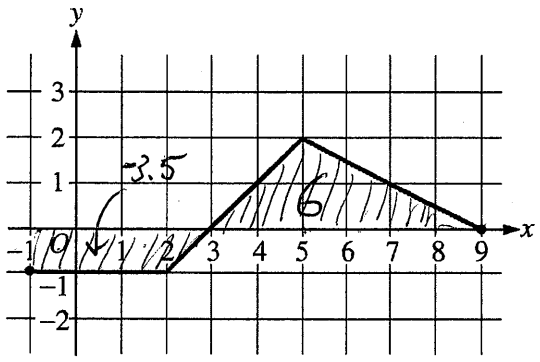
$3\int_{-1}^{9} f(x)\,dx + \int_{-1}^{9} 2\,dx$

$3(2.5) + 2x\Big]_{-1}^{9}$



Graph of f

78. The graph of the piecewise linear function f is shown above. What is the value of $\int_{-1}^{9} (3f(x) + 2)\,dx$?

(A) 7.5 (B) 9.5 (C) 27.5 (D) 47 (E) 48.5

$3(2.5) + 18 - (-2)$

$7.5 + 20 = 27.5$

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79. Let f be a function having derivatives of all orders for $x > 0$ such that $f(3) = 2$, $f'(3) = -1$, $f''(3) = 6$, and $f'''(3) = 12$. Which of the following is the third-degree Taylor polynomial for f about $x = 3$?

$\sum_{n=0}^{\infty} \frac{f^n(c)}{n!}(x-c)^n$

(A) $2 - x + 6x^2 + 12x^3$

(B) $2 - x + 3x^2 + 2x^3$

(C) $2 - (x-3) + 6(x-3)^2 + 12(x-3)^3$

(D) $2 - (x-3) + 3(x-3)^2 + 4(x-3)^3$

(E) $2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$

$f(x) = \underbrace{2 - 1(x-3)}_{\text{tangent line}} + \frac{f''(3)}{2!}(x-3)^2 + \frac{f'''(3)}{3!}(x-3)^3$

$f(x) = 2 - 1(x-3) + \frac{6}{2}(x-3)^2 + \frac{12}{3!}(x-3)^3$

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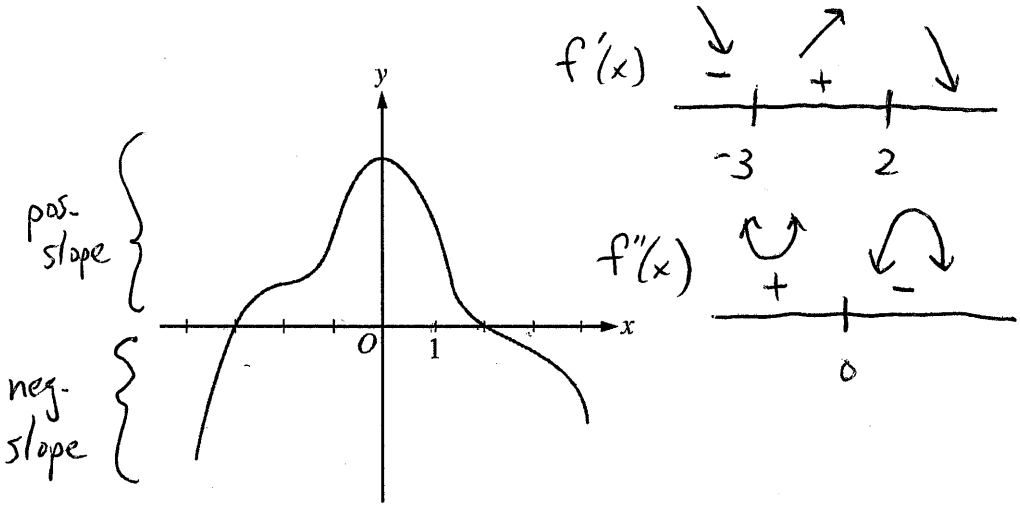


Graph of f'

80. The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

I. f has a relative minimum at $x = -3$.

II. The graph of f has a point of inflection at $x = -2$.

III. The graph of f is concave down for $0 < x < 4$.

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

GO ON TO THE NEXT PAGE.

	$0 < x < 1$	$1 < x < 2$
$f(x)$	Positive	Negative
$f'(x)$	Negative	Negative
$f''(x)$	Negative	Positive

even function

81. Let f be a function that is twice differentiable on $-2 < x < 2$ and satisfies the conditions in the table above. If $f(x) = f(-x)$, what are the x-coordinates of the points of inflection of the graph of f on $-2 < x < 2$?

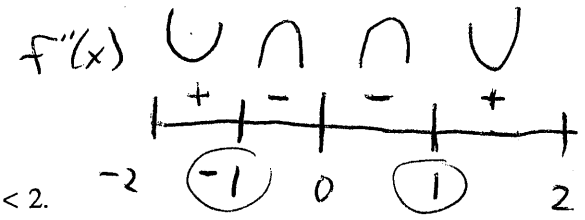
(A) $x = 0$ only

(B) $x = 1$ only

(C) $x = 0$ and $x = 1$

(D) $x = -1$ and $x = 1$

(E) There are no points of inflection on $-2 < x < 2$.



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82. What is the average value of $y = \sqrt{\cos x}$ on the interval $0 \le x \le \frac{\pi}{2}$?

(A) −0.637 (B) 0.500 (C) 0.763 (D) 1.198 (E) 1.882

* Avg. value theorem:

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$\frac{1}{\pi/2-0}\int_0^{\pi/2}\sqrt{\cos x}\,dx = \frac{2}{\pi}\int_0^{\pi/2}\sqrt{\cos x}\,dx$$

$$= 0.763$$

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83. If the function f is continuous at $x = 3$, which of the following must be true?

(A) $f(3) < \lim_{x\to 3} f(x)$ ✗

(B) $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$ ✗

(C) $f(3) = \lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$ ✓

(D) The derivative of f at $x = 3$ exists. ✗ (continuity does not mean differentiable necessarily)

(E) The derivative of f is positive for $x < 3$ and negative for $x > 3$. ✗

→ continuity conditions: i) $f(c)$ exists

ii) $\lim_{x\to c} f(x)$ exists $\left(\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)\right)$

iii) $f(c) = \lim_{x\to c} f(x)$

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84. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{(x^4-2x^2+1)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

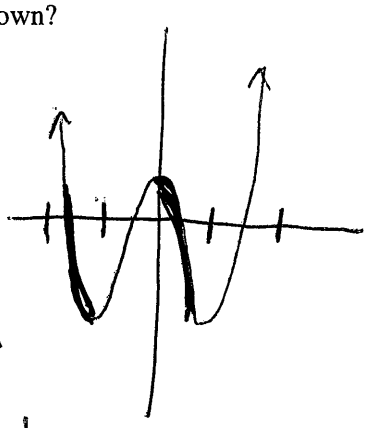
(A) $(-0.418, 0.418)$ only

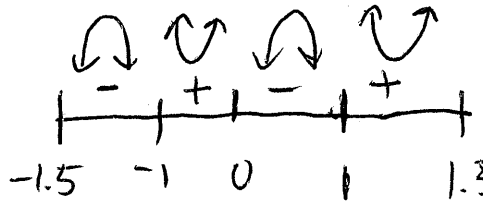
(B) $(-1, 1)$

(C) $(-1.354, -0.409)$ and $(0.409, 1.354)$

(D) $(-1.5, -1)$ and $(0, 1)$

(E) $(-1.5, -1.354)$, $(-0.409, 0)$, and $(1.354, 1.5)$





* $f(x)$ is concave down where $f'(x)$ has negative slope

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85. The fuel consumption of a car, in miles per gallon (mpg), is modeled by $F(s) = 6e^{\left(\frac{s}{20} - \frac{s^2}{2400}\right)}$, where s is the speed of the car, in miles per hour. If the car is traveling at 50 miles per hour and its speed is changing at the rate of 20 miles/hour^2, what is the rate at which its fuel consumption is changing?

(A) 0.215 mpg per hour
(B) 4.299 mpg per hour
(C) 19.793 mpg per hour
(D) 25.793 mpg per hour
(E) 515.855 mpg per hour

$s = 50$ mph

$\frac{ds}{dt} = 20$ m/h^2

$\frac{dF}{dt}$ = Rate that fuel consumption is changing

$$\frac{dF}{dt} = 6e^{\left(\frac{s}{20} - \frac{s^2}{2400}\right)} \cdot \left[\frac{1}{20}\left(\frac{ds}{dt}\right) - \frac{2s}{2400}\left(\frac{ds}{dt}\right)\right]$$

$$\frac{dF}{dt} = 6e^{\left(\frac{50}{20} - \frac{50^2}{2400}\right)}\left[\frac{1}{20} - \frac{2(50)}{2400}\right](20)$$

$$\frac{dF}{dt} = 4.299 \text{ mpg/hr.}$$

* $\frac{d}{dx}e^u = e^u \cdot u'$

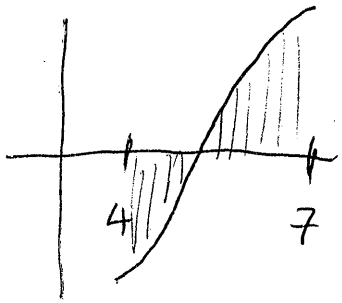
GO ON TO THE NEXT PAGE.

86. If $f'(x) > 0$ for all real numbers x and $\int_4^7 f(t)dt = 0$, which of the following could be a table of values for the function f ?

X (A)

x	$f(x)$
4	−4
5	−3
7	0

$\int f(t)dt < 0$



(B)

x	$f(x)$
4	−4
5	−2
7	5

X (C)

x	$f(x)$
4	−4
5	6
7	3

$f'(x) < 0$

(D)

x	$f(x)$
4	0
5	0
7	0

$f'(x)$ needs to be positive

(E)

x	$f(x)$
4	0
5	4
7	6

portion needs to below x-axis in order for $\int_4^7 f(t)dt = 0$

GO ON TO THE NEXT PAGE.

87. Let R be the region in the first quadrant bounded above by the graph of $y = \ln(3 - x)$, for $0 \le x \le 2$. R is the base of a solid for which each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

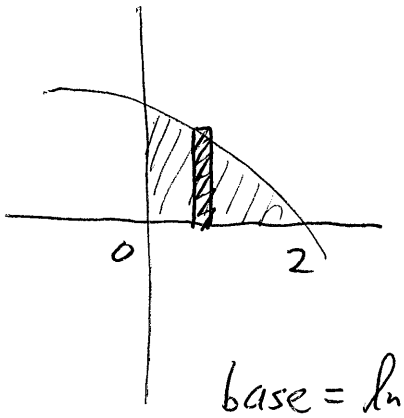
(A) 0.442 (B) 1.029 (C) 1.296 (D) 3.233 (E) 4.071

$\ln(3-x) = 0$

$e^{\ln(3-x)} = e^0$

$3 - x = 1$

$2 = x$



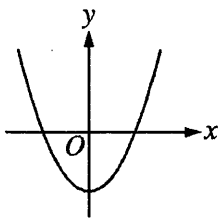
$\text{base} = \ln(3-x) - 0$

$\text{Area}_{\text{square}} = (\text{base})^2$

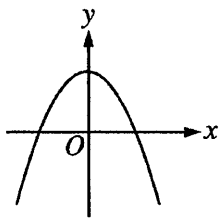
$V = \int_0^2 [\ln(3-x)]^2 \, dx = 1.029$

88. The derivative of a function f is increasing for $x < 0$ and decreasing for $x > 0$. Which of the following could be the graph of f?

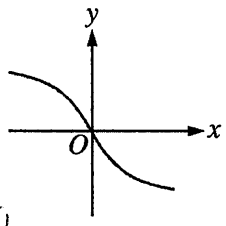
(A)



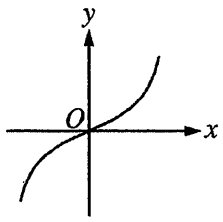
(B)



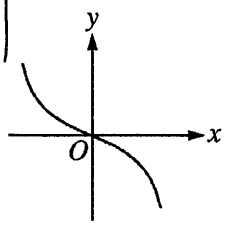
(C)



(D)



(E)



* For $x < 0$, the slope of f is negative but becoming less steep and therefore increasing, moving towards positive slope.
For $x > 0$, f has negative slope and becoming more negative.

89. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \frac{t+3}{\sqrt{t^3+1}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?

(A) 0.713 (B) 1.134 (C) 6.134 (D) 6.710 (E) 11.710

final velocity = initial + displacement of velocity

$$v(3) = v(0) + \int_0^3 a(t)\,dt$$

$$v(3) = 5 + \int_0^3 a(t)\,dt = 11.710$$

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90. If the series $\sum_{n=1}^{\infty} a_n$ converges and $a_n > 0$ for all n, which of the following must be true?

(A) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 0$

(B) $|a_n| < 1$ for all n

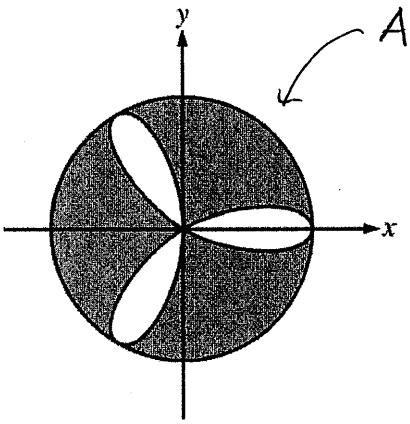
(C) $\sum_{n=1}^{\infty} a_n = 0$

(D) $\sum_{n=1}^{\infty} n a_n$ diverges.

(E) $\sum_{n=1}^{\infty} \frac{a_n}{n}$ converges.

Direct Comparison Test

Since a_n converges and $\frac{a_n}{n} < a_n$, therefore $\frac{a_n}{n}$ must converge as well.



Area of Circle $= \pi r^2$
$= \pi(2)^2$
$= 4\pi$

91. The figure above shows the graphs of the polar curves $r = 2\cos(3\theta)$ and $r = 2$. What is the sum of the areas of the shaded regions?

(A) 0.858 (B) 3.142 (C) 8.566 (D) 9.425 (E) 15.708

*find polar zeros

$2\cos(3\theta) = 0$

$3\theta = \cos^{-1}(0)$

$3\theta = \pi/2, \frac{3\pi}{2}, \frac{5\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{3\pi}{6}, \frac{5\pi}{6}$

$\theta = \pi/6, \frac{\pi}{2}$

Area of one petal $\Rightarrow A = \frac{1}{2}\int_{\pi/6}^{\pi/2} [2\cos(3\theta)]^2 \, d\theta$

$= \frac{1}{2}(2.094) = 1.047$

$A_{\text{shaded region}} = A_{\text{circle}} - A_{(3\text{ petals})}$

$A_s = 4\pi - 3(1.047)$

$A_s = 9.425$

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92. The function h is differentiable, and for all values of x, $h(x) = h(2 - x)$. Which of the following statements must be true?

X I. $\int_0^2 h(x)\,dx > 0$

II. $h'(1) = 0$

X III. $h'(0) = h'(2) = 1$

(A) I only

(B) II only

(C) III only

(D) II and III only

(E) I, II, and III

$h'(x) = h'(2-x) \cdot (-1)$

$h'(x) = -h'(2-x)$

$h'(1) = -h'(2-1)$

$h'(1) = -h'(1)$ ← The only time this is true is if $h'(1) = 0$

$h'(0) = -h'(2-0)$

$h'(0) = -h'(2) \neq 1$

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND ANSWER QUESTIONS 93–96.

Answer Key for AP Calculus BC
Practice Exam, Section I

Multiple-Choice Questions	
Question #	**Key**
1	E
2	A
3	B
4	A
5	C
6	C
7	A
8	C
9	D
10	E
11	A
12	C
13	C
14	E
15	A
16	C
17	C
18	A
19	C
20	C
21	E
22	D
23	A
24	E
25	B
26	B
27	C
28	D
76	B
77	B
78	C
79	E
80	E
81	D
82	C
83	C
84	D
85	B
86	B
87	B
88	E
89	E
90	E
91	D
92	B