EXERCISE 1.1

1. Insert a rational number between and 2/9 and 3/8 arrange in descending order.
Solution:
Given:
Rational numbers: 2/9 and 3/8
Let us rationalize the numbers,
By taking LCM for denominators 9 and 8 which is 72.
2/9 = (2×8)/(9×8) = 16/72
3/8 = (3×9)/(8×9) = 27/72
Since,
16/72 < 27/72
So, 2/9 < 3/8
The rational number between 2/9 and 3/8 is

$$= \frac{\frac{2}{9}+\frac{3}{8}}{2}$$

$$= \frac{\frac{(2\times8)+(3\times9)}{72}}{2}$$

$$= \frac{16+27}{72\times2}$$

$$= \frac{43}{144}$$

Hence, 3/8 > 43/144 > 2/9
The descending order of the numbers is 3/8, 43/144, 2/9

2. Insert two rational numbers between 1/3 and 1/4 and arrange in ascending order.
Solution:
Given:
The rational numbers 1/3 and ¼
By taking LCM and rationalizing, we get

$$= \frac{\frac{1}{3}+\frac{1}{4}}{2}$$

$$= \frac{\frac{4+3}{12}}{2}$$

$$= \frac{7}{12\times2}$$

= 7/24

Now let us find the rational number between ¼ and 7/24
By taking LCM and rationalizing, we get

$$= \frac{\frac{1}{4} + \frac{7}{24}}{2}$$

$$= \frac{\frac{6+7}{24}}{2}$$

$$= \frac{13}{24 \times 2}$$

= 13/48
So,
The two rational numbers between 1/3 and ¼ are
7/24 and 13/48
Hence, we know that, 1/3 > 7/24 > 13/48 > ¼
The ascending order is as follows: ¼, 13/48, 7/24, 1/3

3. Insert two rational numbers between – 1/3 and – 1/2 and arrange in ascending order.

Solution:
Given:
The rational numbers -1/3 and -1/2
By taking LCM and rationalizing, we get

$$= \frac{\frac{-1}{3} + \frac{-1}{2}}{2}$$

$$= \frac{\frac{-2-3}{6}}{2}$$

$$= \frac{-5}{6 \times 2}$$

= -5/12
So, the rational number between -1/3 and -1/2 is -5/12
-1/3 > -5/12 > -1/2

Now, let us find the rational number between -1/3 and -5/12
By taking LCM and rationalizing, we get

$$= \frac{\frac{-1}{3} + \frac{-5}{12}}{2}$$

$$= \frac{\frac{-4-5}{12}}{2}$$

$$= \frac{\frac{-4-5}{12}}{2}$$

$$= \frac{-9}{12 \times 2}$$

= -9/24

= -3/8

So, the rational number between -1/3 and -5/12 is -3/8

-1/3 > -3/8 > -5/12

Hence, the two rational numbers between -1/3 and -1/2 are

-1/3 > -3/8 > -5/12 > -1/2

The ascending is as follows: -1/2, -5/12, -3/8, -1/3

4. Insert three rational numbers between 1/3 and 4/5, and arrange in descending order.

Solution:

Given:

The rational numbers 1/3 and 4/5

By taking LCM and rationalizing, we get

$$= \frac{\frac{1}{3} + \frac{4}{5}}{2}$$

$$= \frac{\frac{5+12}{15}}{2}$$

$$= \frac{17}{15 \times 2}$$

= 17/30

So, the rational number between 1/3 and 4/5 is 17/30

1/3 < 17/30 < 4/5

Now, let us find the rational numbers between 1/3 and 17/30

By taking LCM and rationalizing, we get

$= \frac{\frac{1}{3}+\frac{17}{30}}{2}$

$= \frac{\frac{10+17}{30}}{2}$

$= \frac{27}{30 \times 2}$

= 27/60

So, the rational number between 1/3 and 17/30 is 27/60

1/3 < 27/60 < 17/30

Now, let us find the rational numbers between 17/30 and 4/5

By taking LCM and rationalizing, we get

$= \frac{\frac{17}{30}+\frac{4}{5}}{2}$

$= \frac{\frac{17+24}{30}}{2}$

$= \frac{41}{30 \times 2}$

= 41/60

So, the rational number between 17/30 and 4/5 is 41/60

17/30 < 41/60 < 4/5

Hence, the three rational numbers between 1/3 and 4/5 are

1/3 < 27/60 < 17/30 < 41/60 < 4/5

The descending order is as follows: 4/5, 41/60, 17/30, 27/60, 1/3

5. Insert three rational numbers between 4 and 4.5.

Solution:

Given:

The rational numbers 4 and 4.5

By rationalizing, we get

= (4 + 4.5)/2

= 8.5 / 2

= 4.25

So, the rational number between 4 and 4.5 is 4.25

4 < 4.25 < 4.5

Now, let us find the rational number between 4 and 4.25
By rationalizing, we get
= (4 + 4.25)/2
= 8.25 / 2
= 4.125
So, the rational number between 4 and 4.25 is 4.125
4 < 4.125 < 4.25

Now, let us find the rational number between 4 and 4.125
By rationalizing, we get
= (4 + 4.125)/2
= 8.125 / 2
= 4.0625
So, the rational number between 4 and 4.125 is 4.0625
4 < 4.0625 < 4.125

Hence, the rational numbers between 4 and 4.5 are
4 < 4.0625 < 4.125 < 4.25 < 4.5
The three rational numbers between 4 and 4.5
4.0625, 4.125, 4.25

6. Find six rational numbers between 3 and 4.
Solution:
Given:
The rational number 3 and 4
So let us find the six rational numbers between 3 and 4,
First rational number between 3 and 4 is
= (3 + 4) / 2
= 7/2

Second rational number between 3 and 7/2 is
= (3 + 7/2) / 2
= (6+7) / (2 × 2) [By taking 2 as LCM]
= 13/4

Third rational number between 7/2 and 4 is
= (7/2 + 4) / 2
= (7+8) / (2 × 2) [By taking 2 as LCM]
= 15/4

Fourth rational number between 3 and 13/4 is
= (3 + 13/4) / 2
= (12+13) / (4 × 2) [By taking 4 as LCM]
= 25/8

Fifth rational number between 13/4 and 7/2 is
= [(13/4) + (7/2)] / 2
= [(13+14)/4] / 2 [By taking 4 as LCM]
= (13 + 14) / (4 × 2)
= 27/8

Sixth rational number between 7/2 and 15/4 is
= [(7/2) + (15/4)] / 2
= [(14 + 15)/4] / 2 [By taking 4 as LCM]
= (14 + 15) / (4 × 2)
= 29/8

Hence, the six rational numbers between 3 and 4 are
25/8, 13/4, 27/8, 7/2, 29/8, 15/4

7. Find five rational numbers between 3/5 and 4/5.
Solution:
Given:
The rational numbers 3/5 and 4/5
Now, let us find the five rational numbers between 3/5 and 4/5
So we need to multiply both numerator and denominator with 5 + 1 = 6
We get,
3/5 = (3 × 6) / (5 × 6) = 18/30

4/5 = (4 × 6) / (5 × 6) = 24/30

Now, we have 18/30 < 19/30 < 20/30 < 21/30 < 22/30 < 23/30 < 24/30
Hence, the five rational numbers between 3/5 and 4/5 are
19/30, 20/30, 21/30, 22/30, 23/30

8. Find ten rational numbers between -2/5 and 1/7.
Solution:
Given:

The rational numbers -2/5 and 1/7
By taking LCM for 5 and 7 which is 35
So, -2/5 = (-2 × 7) / (5 × 7) = -14/35

1/7 = (1 × 5) / (7 × 5) = 5/35

Now, we can insert any10 numbers between -14/35 and 5/35
i.e., -13/35, -12/35, -11/35, -10/35, -9/35, -8/35, -7/35, -6/35, -5/35, -4/35, -3/35, -2/35, -1/35, 1/35, 2/35, 3/35, 4/35

Hence, the ten rational numbers between -2/5 and 1/7 are
-6/35, -5/35, -4/35, -3/35, -2/35, -1/35, 1/35, 2/35, 3/35, 4/35

9. Find six rational numbers between 1/2 and 2/3.
Solution:
Given:
The rational number ½ and 2/3
To make the denominators similar let us take LCM for 2 and 3 which is 6
½ = (1 × 3) / (2 × 3) = 3/6
2/3 = (2 × 2) / (3 × 2) = 4/6

Now, we need to insert six rational numbers, so multiply both numerator and denominator by 6 + 1 = 7
3/6 = (3 × 7) / (6 × 7) = 21/42
4/6 = (4 × 7) / (6 × 7) = 28/42

We know that, 21/42 < 22/42 < 23/42 < 24/42 < 25/42 < 26/42 < 27/42 < 28/42
Hence, the six rational numbers between ½ and 2/3 are
22/42, 23/42, 24/42, 25/42, 26/42, 27/42

EXERCISE 1.2

1. Prove that, √5 is an irrational number.

Solution:

Let us consider √5 be a rational number, then

√5 = p/q, where 'p' and 'q' are integers, q ≠ 0 and p, q have no common factors (except 1).

So,

$5 = p^2 / q^2$

$p^2 = 5q^2$ …. (1)

As we know, '5' divides $5q^2$, so '5' divides p^2 as well. Hence, '5' is prime.

So 5 divides p

Now, let p = 5k, where 'k' is an integer

Square on both sides, we get

$p^2 = 25k^2$

$5q^2 = 25k^2$ [Since, $p^2 = 5q^2$, from equation (1)]

$q^2 = 5k^2$

As we know, '5' divides $5k^2$, so '5' divides q^2 as well. But '5' is prime.

So 5 divides q

Thus, p and q have a common factor 5. This statement contradicts that 'p' and 'q' has no common factors (except 1).

We can say that, √5 is not a rational number.

√5 is an irrational number.

Hence proved.

2. Prove that, √7 is an irrational number.

Solution:

Let us consider √7 be a rational number, then

√7 = p/q, where 'p' and 'q' are integers, q ≠ 0 and p, q have no common factors (except 1).

So,

$7 = p^2 / q^2$

$p^2 = 7q^2$ …. (1)

As we know, '7' divides $7q^2$, so '7' divides p^2 as well. Hence, '7' is prime.

So 7 divides p

Now, let p = 7k, where 'k' is an integer

Square on both sides, we get
$p^2 = 49k^2$
$7q^2 = 49k^2$ [Since, $p^2 = 7q^2$, from equation (1)]
$q^2 = 7k^2$

As we know, '7' divides $7k^2$, so '7' divides q^2 as well. But '7' is prime.
So 7 divides q
Thus, p and q have a common factor 7. This statement contradicts that 'p' and 'q' has no common factors (except 1).
We can say that, √7 is not a rational number.
√7 is an irrational number.
Hence proved.

3. Prove that √6 is an irrational number.
Solution:
Let us consider √6 be a rational number, then
√6 = p/q, where 'p' and 'q' are integers, $q \neq 0$ and p, q have no common factors (except 1).
So,
$6 = p^2 / q^2$
$p^2 = 6q^2$ …. (1)

As we know, '2' divides $6q^2$, so '2' divides p^2 as well. Hence, '2' is prime.
So 2 divides p
Now, let p = 2k, where 'k' is an integer
Square on both sides, we get
$p^2 = 4k^2$
$6q^2 = 4k^2$ [Since, $p^2 = 6q^2$, from equation (1)]
$3q^2 = 2k^2$

As we know, '2' divides $2k^2$, so '2' divides $3q^2$ as well.
'2' should either divide 3 or divide q^2.
But '2' does not divide 3. '2' divides q^2 so '2' is prime.
So 2 divides q
Thus, p and q have a common factor 2. This statement contradicts that 'p' and 'q' has no common factors (except 1).
We can say that, √6 is not a rational number.
√6 is an irrational number.
Hence proved.

4. Prove that 1/√11 is an irrational number.

Solution:

Let us consider 1/√11 be a rational number, then

1/√11 = p/q, where ‘p’ and ‘q’ are integers, q ≠ 0 and p, q have no common factors (except 1).

So,

$1/11 = p^2 / q^2$

$q^2 = 11p^2$ …. (1)

As we know, ‘11’ divides $11p^2$, so ‘11’ divides q^2 as well. Hence, ‘11’ is prime.

So 11 divides q

Now, let q = 11k, where ‘k’ is an integer

Square on both sides, we get

$q^2 = 121k^2$

$11p^2 = 121k^2$ [Since, $q^2 = 11p^2$, from equation (1)]

$p^2 = 11k^2$

As we know, ‘11’ divides $11k^2$, so ‘11’ divides p^2 as well. But ‘11’ is prime.

So 11 divides p

Thus, p and q have a common factor 11. This statement contradicts that ‘p’ and ‘q’ has no common factors (except 1).

We can say that, 1/√11 is not a rational number.

1/√11 is an irrational number.

Hence proved.

5. Prove that √2 is an irrational number. Hence show that 3 — √2 is an irrational.

Solution:

Let us consider √2 be a rational number, then

√2 = p/q, where ‘p’ and ‘q’ are integers, q ≠ 0 and p, q have no common factors (except 1).

So,

$2 = p^2 / q^2$

$p^2 = 2q^2$ …. (1)

As we know, ‘2’ divides $2q^2$, so ‘2’ divides p^2 as well. Hence, ‘2’ is prime.

So 2 divides p

Now, let p = 2k, where ‘k’ is an integer

Square on both sides, we get

$p^2 = 4k^2$
$2q^2 = 4k^2$ [Since, $p^2 = 2q^2$, from equation (1)]
$q^2 = 2k^2$

As we know, '2' divides $2k^2$, so '2' divides q^2 as well. But '2' is prime.
So 2 divides q
Thus, p and q have a common factor 2. This statement contradicts that 'p' and 'q' has no common factors (except 1).
We can say that, √2 is not a rational number.
√2 is an irrational number.

Now, let us assume 3 - √2 be a rational number, 'r'
So, 3 - √2 = r
3 – r = √2
We know that, 'r' is rational, '3- r' is rational, so '√2' is also rational.
This contradicts the statement that √2 is irrational.
So, 3- √2 is irrational number.
Hence proved.

6. Prove that, √3 is an irrational number. Hence, show that 2/5×√3 is an irrational number.
Solution:
Let us consider √3 be a rational number, then
√3 = p/q, where 'p' and 'q' are integers, q ≠ 0 and p, q have no common factors (except 1).
So,
$3 = p^2 / q^2$
$p^2 = 3q^2$ …. (1)

As we know, '3' divides $3q^2$, so '3' divides p^2 as well. Hence, '3' is prime.
So 3 divides p
Now, let p = 3k, where 'k' is an integer
Square on both sides, we get
$p^2 = 9k^2$
$3q^2 = 9k^2$ [Since, $p^2 = 3q^2$, from equation (1)]
$q^2 = 3k^2$

As we know, '3' divides $3k^2$, so '3' divides q^2 as well. But '3' is prime.
So 3 divides q

Thus, p and q have a common factor 3. This statement contradicts that ‘p’ and ‘q’ has no common factors (except 1).
We can say that, $\sqrt{3}$ is not a rational number.
$\sqrt{3}$ is an irrational number.

Now, let us assume $(2/5)\sqrt{3}$ be a rational number, ‘r’
So, $(2/5)\sqrt{3} = r$
$5r/2 = \sqrt{3}$
We know that, ‘r’ is rational, ‘5r/2’ is rational, so ‘$\sqrt{3}$’ is also rational.
This contradicts the statement that $\sqrt{3}$ is irrational.
So, $(2/5)\sqrt{3}$ is irrational number.
Hence proved.

7. Prove that $\sqrt{5}$ is an irrational number. Hence, show that $-3 + 2\sqrt{5}$ is an irrational number.
Solution:
Let us consider $\sqrt{5}$ be a rational number, then
$\sqrt{5} = p/q$, where ‘p’ and ‘q’ are integers, $q \neq 0$ and p, q have no common factors (except 1).
So,
$5 = p^2 / q^2$
$p^2 = 5q^2$ …. (1)

As we know, ‘5’ divides $5q^2$, so ‘5’ divides p^2 as well. Hence, ‘5’ is prime.
So 5 divides p
Now, let $p = 5k$, where ‘k’ is an integer
Square on both sides, we get
$p^2 = 25k^2$
$5q^2 = 25k^2$ [Since, $p^2 = 5q^2$, from equation (1)]
$q^2 = 5k^2$

As we know, ‘5’ divides $5k^2$, so ‘5’ divides q^2 as well. But ‘5’ is prime.
So 5 divides q
Thus, p and q have a common factor 5. This statement contradicts that ‘p’ and ‘q’ has no common factors (except 1).
We can say that, $\sqrt{5}$ is not a rational number.
$\sqrt{5}$ is an irrational number.

Now, let us assume $-3 + 2\sqrt{5}$ be a rational number, ‘r’

So, -3 + 2√5 = r
-3 - r = 2√5
(-3 - r)/2 = √5
We know that, ‘r’ is rational, ‘(-3 - r)/2’ is rational, so ‘√5’ is also rational.
This contradicts the statement that √5 is irrational.
So, -3 + 2√5 is irrational number.
Hence proved.

8. Prove that the following numbers are irrational:
(i) 5 +√2
(ii) 3 - 5√3
(iii) 2√3 - 7
(iv) √2 +√5
Solution:
(i) 5 +√2
Now, let us assume 5 + √2 be a rational number, ‘r’
So, 5 + √2 = r
r - 5 = √2
We know that, ‘r’ is rational, ‘r - 5’ is rational, so ‘√2’ is also rational.
This contradicts the statement that √2 is irrational.
So, 5 + √2 is irrational number.

(ii) 3 - 5√3
Now, let us assume 3 - 5√3 be a rational number, ‘r’
So, 3 - 5√3 = r
3 - r = 5√3
(3 - r)/5 = √3
We know that, ‘r’ is rational, ‘(3 - r)/5’ is rational, so ‘√3’ is also rational.
This contradicts the statement that √3 is irrational.
So, 3 - 5√3 is irrational number.

(iii) 2√3 - 7
Now, let us assume 2√3 – 7 be a rational number, ‘r’
So, 2√3 – 7 = r
2√3 = r + 7
√3 = (r + 7)/2
We know that, ‘r’ is rational, ‘(r + 7)/2’ is rational, so ‘√3’ is also rational.
This contradicts the statement that √3 is irrational.
So, 2√3 – 7 is irrational number.

(iv) $\sqrt{2} + \sqrt{5}$

Now, let us assume $\sqrt{2} + \sqrt{5}$ be a rational number, 'r'

So, $\sqrt{2} + \sqrt{5} = r$

$\sqrt{5} = r - \sqrt{2}$

Square on both sides,

$(\sqrt{5})^2 = (r - \sqrt{2})^2$

$5 = r^2 + (\sqrt{2})^2 – 2r\sqrt{2}$

$5 = r^2 + 2 - 2\sqrt{2}r$

$5 – 2 = r^2 - 2\sqrt{2}r$

$r^2 – 3 = 2\sqrt{2}r$

$(r^2 - 3)/2r = \sqrt{2}$

We know that, 'r' is rational, '$(r^2 - 3)/2r$' is rational, so '$\sqrt{2}$' is also rational.

This contradicts the statement that $\sqrt{2}$ is irrational.

So, $\sqrt{2} + \sqrt{5}$ is irrational number.

EXERCISE 1.3

1. Locate √10 and √17 on the amber line.

Solution:

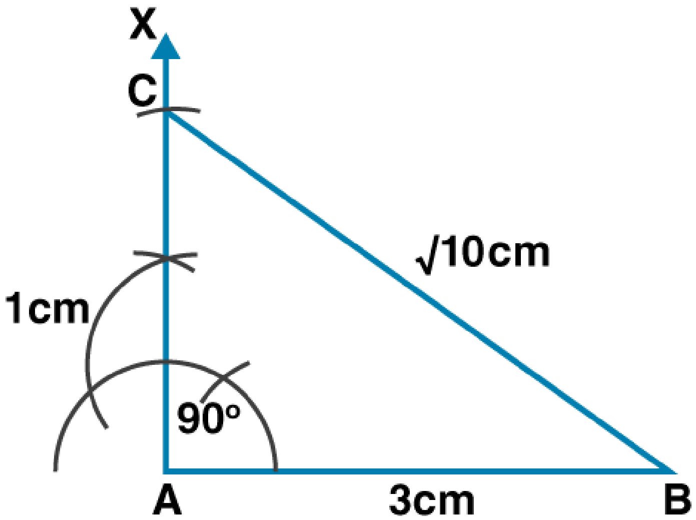
√10

$\sqrt{10} = \sqrt{(9 + 1)} = \sqrt{((3)^2 + 1^2)}$

Now let us construct:

- Draw a line segment AB = 3cm.
- At point A, draw a perpendicular AX and cut off AC = 1cm.
- Join BC.

  BC = √10cm



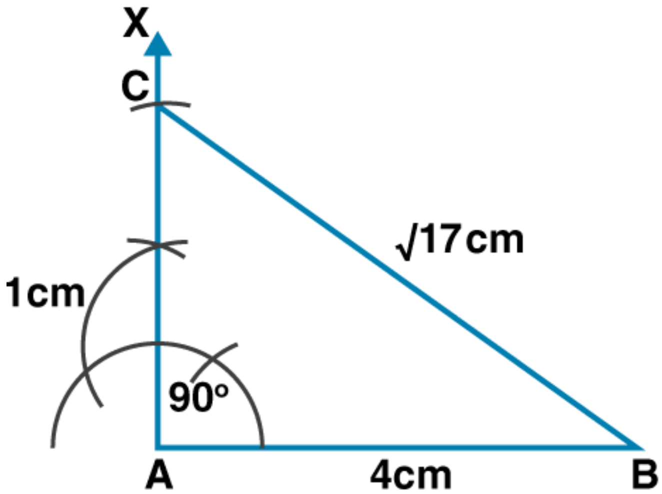
√17

$\sqrt{17} = \sqrt{(16 + 1)} = \sqrt{((4)^2 + 1^2)}$

Now let us construct:

- Draw a line segment AB = 4cm.
- At point A, draw a perpendicular AX and cut off AC = 1cm.
- Join BC.

  BC = √17cm



2. Write the decimal expansion of each of the following numbers and say what kind of decimal expansion each has:

(i) 36/100

(ii) 4 1/8

(iii) 2/9

(iv) 2/11

(v) 3/13

(vi) 329/400

Solution:

(i) 36/100

			0	0.	3	6	0
1	0	0	3	6.	0	0	0
		–	0				
			3	6			
		–		0			
			3	6	0		
		–	3	0	0		
				6	0	0	
			–	6	0	0	
						0	0
					–		0
							0

36/100 = 0.36
It is a terminating decimal.

(ii) 4 1/8
4 1/8 = (4×8 + 1)/8 = 33/8

	0	4.	1	2	5
8	3	3.	0	0	0
–	0				
	3	3			
–	3	2			
		1	0		
	–		8		
			2	0	
		–	1	6	
				4	0
			–	4	0
					0

33/8 = 4.125
It is a terminating decimal.

(iii) 2/9

	0.	2	2	2
9	2.	0	0	0
–	0			
	2	0		
–	1	8		
		2	0	
	–	1	8	
			2	0
		–	1	8
				2

2/9 = 0.222
It is a non-terminating recurring decimal.

(iv) 2/11

		0.	1	8	1
1	1	2.	0	0	0
	−	0			
		2	0		
	−	1	1		
			9	0	
		−	8	8	
				2	0
			−	1	1
					9

2/11 = 0.181
It is a non-terminating recurring decimal.

(v) 3/13

		0.	2	3	0	7	6	9	2	3	0	7
1	3	3.	0	0	0	0	0	0	0	0	0	0
	−	0										
		3	0									
	−	2	6									
			4	0								
		−	3	9								
				1	0							
			−		0							
				1	0	0						
			−		9	1						
						9	0					
					−	7	8					
						1	2	0				
					−	1	1	7				
								3	0			
							−	2	6			
									4	0		
								−	3	9		
										1	0	
									−		0	
										1	0	0
									−		9	1
												9

3/13 = 0.2317692307
It is a non-terminating recurring decimal.

(vi) 329/400

			0	0	0.	8	2	2	5
4	0	0	3	2	9.	0	0	0	0
		−	0						
			3	2					
		−		0					
			3	2	9				
		−			0				
			3	2	9	0			
		−	3	2	0	0			
					9	0	0		
				−	8	0	0		
					1	0	0	0	
				−		8	0	0	
						2	0	0	0
					−	2	0	0	0
									0

329/400 = 0.8225
It is a terminating decimal.

3. Without actually performing the king division, State whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:
(i) 13/3125
(ii) 17/8
(iii) 23/75
(iv) 6/15
(v) 1258/625
(vi) 77/210
Solution:
We know that, if the denominator of a fraction has only 2 or 5 or both factors, it is a terminating decimal otherwise it is non-terminating repeating decimals.
(i) 13/3125

5	3125
5	625
5	125
5	25
5	5
	1

3125 = 5 × 5 × 5 × 5 × 5
Prime factor of 3125 = 5, 5, 5, 5, 5 [i.e., in the form of 2^n, 5^n]
It is a terminating decimal.

(ii) 17/8

2	8
2	4
2	2
	1

8 = 2 × 2 × 2
Prime factor of 8 = 2, 2, 2 [i.e., in the form of 2^n, 5^n]
It is a terminating decimal.

(iii) 23/75

3	75
5	25
5	5
	1

75 = 3 × 5 × 5
Prime factor of 75 = 3, 5, 5
It is a non-terminating repeating decimal.

(iv) 6/15
Let us divide both numerator and denominator by 3
6/15 = (6 ÷ 3) / (15 ÷ 3)
= 2/5
Since the denominator is 5.
It is a terminating decimal.

(v) 1258/625

5	625
5	125
5	25
5	5
	1

625 = 5 × 5 × 5 × 5
Prime factor of 625 = 5, 5, 5, 5 [i.e., in the form of 2^n, 5^n]
It is a terminating decimal.

(vi) 77/210
Let us divide both numerator and denominator by 7

77/210 = (77 ÷ 7) / (210 ÷ 7)
= 11/30

2	30
3	15
5	5
	1

30 = 2 × 3 × 5
Prime factor of 30 = 2, 3, 5
It is a non-terminating repeating decimal.

4. Without actually performing the long division, find if 987/10500 will have terminating or non-terminating repeating decimal expansion. Give reasons for your answer.
Solution:
Given:
The fraction 987/10500
Let us divide numerator and denominator by 21, we get
987/10500 = (987 ÷ 21) / (10500 ÷ 21)
= 47/500
So,
The prime factors for denominator 500 = 2 × 2 × 5 × 5 × 5
Since it is of the form: 2^n, 5^n
Hence it is a terminating decimal.

5. Write the decimal expansions of the following numbers which have terminating decimal expansions:
(i) 17/8
(ii) 13/3125
(iii) 7/80
(iv) 6/15
(v) $2^2 \times 7/5^4$
(vi) 237/1500
Solution:
(i) 17/8

2	8
2	4
2	2
	1

Denominator, 8 = 2 × 2 × 2
= 2^3

It is a terminating decimal.

When we divide 17/8, we get

	0	2.	1	2	5	0
8	1	7.	0	0	0	0
−	0					
	1	7				
−	1	6				
		1	0			
	−		8			
			2	0		
		−	1	6		
				4	0	
			−	4	0	
					0	0
				−		0
						0

17/8 = 2.125

(ii) 13/3125

5	3125
5	625
5	125
5	25
5	5
	1

3125 = 5 × 5 × 5 × 5 × 5

Prime factor of 3125 = 5, 5, 5, 5, 5 [i.e., in the form of 2^n, 5^n]

It is a terminating decimal.

When we divide 13/3125, we get

				0	0.	0	0	4	1	6
3	1	2	5	1	3.	0	0	0	0	0
			–	0						
				1	3					
			–		0					
				1	3	0				
			–			0				
				1	3	0	0			
			–				0			
				1	3	0	0	0		
			–	1	2	5	0	0		
						5	0	0	0	
					–	3	1	2	5	
						1	8	7	5	0
					–	1	8	7	5	0
										0

13/3125 = 0.00416

(iii) 7/80

2	80
2	40
2	20
2	10
5	5
	1

80 = 2 × 2 × 2 × 2 × 5

Prime factor of 80 = 2^4, 5^1 [i.e., in the form of 2^n, 5^n]

It is a terminating decimal.

When we divide 7/80, we get

		0.	0	8	7	5
8	0	7.	0	0	0	0
	–	0				
		7	0			
	–		0			
		7	0	0		
	–	6	4	0		
			6	0	0	
		–	5	6	0	
				4	0	0
			–	4	0	0
						0

7/80 = 0.0875

(iv) 6/15

Let us divide both numerator and denominator by 3, we get

6/15 = (6 ÷ 3) / (15 ÷ 3)

= 2/5

Since the denominator is 5,

It is terminating decimal.

		0.	4	0
1	5	6.	0	0
	–	0		
		6	0	
	–	6	0	
			0	0
		–		0
				0

6/15 = 0.4

(v) $(2^2 \times 7)/5^4$

We know that the denominator is 5^4

It is a terminating decimal.

$(2^2 \times 7)/5^4 = (2 \times 2 \times 7) / (5 \times 5 \times 5 \times 5)$

$= 28/625$

			0	0.	0	4	4	8
6	2	5	2	8.	0	0	0	0
		–	0					
			2	8				
		–		0				
			2	8	0			
		–			0			
			2	8	0	0		
		–	2	5	0	0		
				3	0	0	0	
			–	2	5	0	0	
					5	0	0	0
				–	5	0	0	0
								0

28/625 = 0.0448
It is a terminating decimal.

(vi) 237/1500
Let us divide both numerator and denominator by 3, we get
237/1500 = (237 ÷ 3) / (1500 ÷ 3)
= 79/500
Since the denominator is 500,
Its factors are, 500 = 2 × 2 × 5 × 5 × 5
$= 2^2 \times 5^3$
It is terminating decimal.

			0	0.	1	5	8
5	0	0	7	9.	0	0	0
		–	0				
			7	9			
		–		0			
			7	9	0		
		–	5	0	0		
			2	9	0	0	
		–	2	5	0	0	
				4	0	0	0
			–	4	0	0	0
							0

237/1500= 79/500 = 0.1518

6. Write the denominator of the rational number 257/5000 in the form $2^m \times 5^n$ where m, n is non-negative integers. Hence, write its decimal expansion on without actual division.

Solution:

Given:

The fraction 257/5000

Since the denominator is 5000,

The factors for 5000 are:

2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

$5000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$

$= 2^3 \times 5^4$

$257/5000 = 257/(2^3 \times 5^4)$

It is a terminating decimal.

So,

Let us multiply both numerator and denominator by 2, we get

$257/5000 = (257 \times 2) / (5000 \times 2)$

$= 514/10000$

$= 0.0514$

7. Write the decimal expansion of 1/7. Hence, write the decimal expression of? 2/7, 3/7, 4/7, 5/7 and 6/7.

Solution:

Given:

The fraction: 1/7

	0.	1	4	2	8	5	7	1	4	2	8	5	7
7	1.	0	0	0	0	0	0	0	0	0	0	0	0
−	0												
	1	0											
−		7											
		3	0										
	−	2	8										
			2	0									
		−	1	4									
				6	0								
			−	5	6								
					4	0							
				−	3	5							
						5	0						
					−	4	9						
							1	0					
						−		7					
								3	0				
							−	2	8				
									2	0			
								−	1	4			
										6	0		
									−	5	6		
											4	0	
										−	3	5	
												5	0
											−	4	9
													1

1/7 = 0.142857142857

Since it is recurring,

$$= 0.\overline{142857}$$

Now,

2/7 = 2 × (1/7)

$= 2 \times 0.\overline{142857}$

$= 0.\overline{285714}$

3/7 = 3 × (1/7)

$= 3 \times 0.\overline{142857}$

$= 0.\overline{428571}$

4/7 = 4 × (1/7)
$= 4 \times 0.\overline{142857}$
$= 0.\overline{571428}$

5/7 = 5 × (1/7)
$= 5 \times 0.\overline{142857}$
$= 0.\overline{714285}$

6/7 = 6 × (1/7)
$= 6 \times 0.\overline{142857}$
$= 0.\overline{857142}$

8. Express the following numbers in the form p/q'. Where p and q are both integers and q≠0;

$(i) 0.\overline{3}$

$(ii) 5.\overline{2}$

$(iii) 0.404040....$

$(iv) 0.4\overline{7}$

$(v) 0.1\overline{34}$

$(vi) 0.\overline{001}$

Solution:

$(i) 0.\overline{3}$

Let x = $0.\overline{3}$ = 0.3333…
Since there is one repeating digit after the decimal point,
Multiplying by 10 on both sides, we get
10x = 3.3333…
Now, subtract both the values,
9x = 3
x = 3/9
= 1/3
$0.\overline{3}$ = 1/3

$(ii) 5.\overline{2}$

Let x = $5.\overline{2}$ = 5.2222…
Since there is one repeating digit after the decimal point,

Multiplying by 10 on both sides, we get
10x = 52.2222…
Now, subtract both the values,
9x = 52 – 5
9x = 47
x = 47/9
$5.\overline{2}$ = 47/9

$(iii) 0.404040....$
Let x = 0.404040
Since there is two repeating digit after the decimal point,
Multiplying by 100 on both sides, we get
100x = 40.404040…
Now, subtract both the values,
99x = 40
x = 40/99
0.404040… = 40/99

$(iv) 0.4\overline{7}$
Let x = $0.4\overline{7}$ = 0.47777…
Since there is one non-repeating digit after the decimal point,
Multiplying by 10 on both sides, we get
10x = 4.7777
Since there is one repeating digit after the decimal point,
Multiplying by 10 on both sides, we get
100x = 47.7777
Now, subtract both the values,
90x = 47 – 4
90x = 43
x = 43/90
$0.4\overline{7}$ = 43/90

$(v) 0.1\overline{34}$
Let x = $0.1\overline{34}$ = 0.13434343…
Since there is one non-repeating digit after the decimal point,
Multiplying by 10 on both sides, we get
10x = 1.343434
Since there is two repeating digit after the decimal point,

Multiplying by 100 on both sides, we get
1000x = 134.343434
Now, subtract both the values,
990x = 133
x = 133/990
$0.1\overline{34}$ = 133/990

$(vi) 0.\overline{001}$
Let x = $0.\overline{001}$ = 0.001001001…
Since there is three repeating digit after the decimal point,
Multiplying by 1000 on both sides, we get
1000x = 1.001001
Now, subtract both the values,
999x = 1
x = 1/999
$0.\overline{001}$ = 1/999

9. Classify the following numbers as rational or irrational:
(i) √23
(ii) √225
(iii) 0.3796
(iv) 7.478478
(v) 1.101001000100001…
$(vi) 345.0\overline{456}$
Solution:
(i) √23
Since, 23 is not a perfect square,
√23 is an irrational number.

(ii) √225
$\sqrt{225} = \sqrt{(15)^2} = 15$
Since, 225 is a perfect square,
√225 is a rational number.

(iii) 0.3796
0.3796 = 3796/1000
Since, the decimal expansion is terminating decimal.
0.3796 is a rational number.

(iv) 7.478478
Let x = 7.478478
Since there is three repeating digit after the decimal point,
Multiplying by 1000 on both sides, we get
1000x = 7478.478478…
Now, subtract both the values,
999x = 7478 – 7
999x = 7471
x = 7471/999
7.478478 = 7471/999
Hence, it is neither terminating nor non-terminating or non-repeating decimal.
7.478478 is an irrational number.

(v) 1.101001000100001…
Since number of zero's between two consecutive ones are increasing. So it is non-terminating or non-repeating decimal.
1.101001000100001… is an irrational number.

$(vi) 345.0\overline{456}$
Let x = 345.0456456
Multiplying by 10 on both sides, we get
10x = 3450.456456
Since there is three repeating digit after the decimal point,
Multiplying by 1000 on both sides, we get
1000x = 3450456.456456…
Now, subtract both the values,
10000x – 10x = 3450456 – 345
9990x = 3450111
x = 3450111/9990
Since, it is non-terminating repeating decimal.
$345.0\overline{456}$ is a rational number.

10. Insert… following.
(i) One irrational number between 1/3 and ½
(ii) One irrational number between -2/5 and ½
(iii) One irrational number between 0 and 0.1
Solution:
(i) One irrational number between 1/3 and ½

	0.	3	3	3
3	1.	0	0	0
–	0			
	1	0		
–		9		
		1	0	
	–		9	
			1	0
		–		9
				1

1/3 = 0.333…

	0.	5
2	1.	0
–	0	
	1	0
–	1	0
		0

½ = 0.5

So there are infinite irrational numbers between 1/3 and ½.

One irrational number among them can be 0.4040040004…

(ii) One irrational number between -2/5 and ½

		-	0.	4
+	5	-	2.	0
	–		0	
			2	0
		–	2	0
				0

-2/5 = -0.4

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	0.	5
2	1.	0
−	0	
	1	0
−	1	0
		0

½ = 0.5
So there are infinite irrational numbers between -2/5 and ½.
One irrational number among them can be 0.1010010001…

(iii) One irrational number between 0 and 0.1
There are infinite irrational numbers between 0 and 1.
One irrational number among them can be 0.06006000600006…

11. Insert two irrational numbers between 2 and 3.
Solution:
2 is expressed as √4
And 3 is expressed as √9
So, two irrational numbers between 2 and 3 or √4 and √9 are √5, √6

12. Write two irrational numbers between 4/9 and 7/11.
Solution:
4/9 is expressed as 0.4444…
7/11 is expressed as 0.636363…
So, two irrational numbers between 4/9 and 7/11 are 0.4040040004… and 0.6060060006…

13. Find one rational number between √2 and √3.
Solution:
√2 is expressed as 1.4142…
√3 is expressed as 1.7320…
So, one rational number between √2 and √3 is 1.5.

14. Find two rational numbers between √12 and √15.
Solution:
√12 = √(4×3) = 2√3
Since, 12 < 12.25 < 12.96 < 15

So, $\sqrt{12} < \sqrt{12.25} < \sqrt{12.96} < \sqrt{15}$

Hence, two rational numbers between $\sqrt{12}$ and $\sqrt{15}$are [$\sqrt{12.25}$, $\sqrt{12.96}$] or [$\sqrt{3.5}$, $\sqrt{3.6}$].

15. Insert irrational numbers between $\sqrt{5}$ and $\sqrt{7}$.

Solution:

Since, $5 < 6 < 7$

So, irrational number between $\sqrt{5}$ and $\sqrt{7}$ is $\sqrt{6}$.

16. Insert two irrational numbers between $\sqrt{3}$ and $\sqrt{7}$.

Solution:

Since, $3 < 4 < 5 < 6 < 7$

So,

$\sqrt{3} < \sqrt{4} < \sqrt{5} < \sqrt{6} < \sqrt{7}$

But $\sqrt{4} = 2$, which is a rational number.

So,

Two irrational numbers between $\sqrt{3}$ and $\sqrt{7}$ are $\sqrt{5}$ and $\sqrt{6}$.

EXERCISE 1.4

1. Simplify the following:
(i) √45 - 3√20 + 4√5
(ii) 3√3 + 2√27 + 7/√3
(iii) 6√5 × 2√5
(iv) 8√15 ÷ 2√3
(v) √24/8 + √54/9
(vi) 3/√8 + 1/√2
Solution:
(i) √45 - 3√20 + 4√5
Let us simplify the expression,
√45 - 3√20 + 4√5
= √(9×5) - 3√(4×5) + 4√5
= 3√5 - 3×2√5 + 4√5
= 3√5 - 6√5 + 4√5
= √5

(ii) 3√3 + 2√27 + 7/√3
Let us simplify the expression,
3√3 + 2√27 + 7/√3
= 3√3 + 2√(9×3) + 7√3/(√3×√3) (by rationalizing)
= 3√3 + (2×3)√3 + 7√3/3
= 3√3 + 6√3 + (7/3) √3
= √3 (3 + 6 + 7/3)
= √3 (9 + 7/3)
= √3 (27+7)/3
= 34/3 √3

(iii) 6√5 × 2√5
Let us simplify the expression,
6√5 × 2√5
= 12 × 5
= 60

(iv) 8√15 ÷ 2√3
Let us simplify the expression,
8√15 ÷ 2√3
= (8 √5 √3) / 2√3

= $4\sqrt{5}$

(v) $\sqrt{24}/8 + \sqrt{54}/9$
Let us simplify the expression,
$\sqrt{24}/8 + \sqrt{54}/9$
$= \sqrt{(4\times6)}/8 + \sqrt{(9\times6)}/9$
$= 2\sqrt{6}/8 + 3\sqrt{6}/9$
$= \sqrt{6}/4 + \sqrt{6}/3$
By taking LCM
$= (3\sqrt{6} + 4\sqrt{6})/12$
$= 7\sqrt{6}/12$

(vi) $3/\sqrt{8} + 1/\sqrt{2}$
Let us simplify the expression,
$3/\sqrt{8} + 1/\sqrt{2}$
$= 3/2\sqrt{2} + 1/\sqrt{2}$
By taking LCM
$= (3 + 2)/(2\sqrt{2})$
$= 5/(2\sqrt{2})$
By rationalizing,
$= 5\sqrt{2}/(2\sqrt{2} \times 2\sqrt{2})$
$= 5\sqrt{2}/(2\times2)$
$= 5\sqrt{2}/4$

2. Simplify the following:
(i) $(5 + \sqrt{7})(2 + \sqrt{5})$
(ii) $(5 + \sqrt{5})(5 - \sqrt{5})$
(iii) $(\sqrt{5} + \sqrt{2})^2$
(iv) $(\sqrt{3} - \sqrt{7})^2$
(v) $(\sqrt{2} + \sqrt{3})(\sqrt{5} + \sqrt{7})$
(vi) $(4 + \sqrt{5})(\sqrt{3} - \sqrt{7})$
Solution:
(i) $(5 + \sqrt{7})(2 + \sqrt{5})$
Let us simplify the expression,
$= 5(2 + \sqrt{5}) + \sqrt{7}(2 + \sqrt{5})$
$= 10 + 5\sqrt{5} + 2\sqrt{7} + \sqrt{35}$

(ii) $(5 + \sqrt{5})(5 - \sqrt{5})$
Let us simplify the expression,

By using the formula,
$(a)^2 – (b)^2 = (a + b) (a - b)$
So,
$= (5)^2 – (\sqrt{5})^2$
$= 25 – 5$
$= 20$

(iii) $(\sqrt{5} + \sqrt{2})^2$
Let us simplify the expression,
By using the formula,
$(a + b)^2 = a^2 + b^2 + 2ab$
$(\sqrt{5} + \sqrt{2})^2 = (\sqrt{5})^2 + (\sqrt{2})^2 + 2\sqrt{5}\sqrt{2}$
$= 5 + 2 + 2\sqrt{10}$
$= 7 + 2\sqrt{10}$

(iv) $(\sqrt{3} - \sqrt{7})^2$
Let us simplify the expression,
By using the formula,
$(a - b)^2 = a^2 + b^2 - 2ab$
$(\sqrt{3} - \sqrt{7})^2 = (\sqrt{3})^2 + (\sqrt{7})^2 - 2\sqrt{3}\sqrt{7}$
$= 3 + 7 - 2\sqrt{21}$
$= 10 - 2\sqrt{21}$

(v) $(\sqrt{2} + \sqrt{3}) (\sqrt{5} + \sqrt{7})$
Let us simplify the expression,
$= \sqrt{2}(\sqrt{5} + \sqrt{7}) + \sqrt{3}(\sqrt{5} + \sqrt{7})$
$= \sqrt{2}\times\sqrt{5} + \sqrt{2}\times\sqrt{7} + \sqrt{3}\times\sqrt{5} + \sqrt{3}\times\sqrt{7}$
$= \sqrt{10} + \sqrt{14} + \sqrt{15} + \sqrt{21}$

(vi) $(4 + \sqrt{5}) (\sqrt{3} - \sqrt{7})$
Let us simplify the expression,
$= 4(\sqrt{3} - \sqrt{7}) + \sqrt{5}(\sqrt{3} - \sqrt{7})$
$= 4\sqrt{3} - 4\sqrt{7} + \sqrt{15} - \sqrt{35}$

3. If $\sqrt{2} = 1.414$, then find the value of
(i) $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$
(ii) $3\sqrt{32} - 2\sqrt{50} + 4\sqrt{128} - 20\sqrt{18}$
Solution:
(i) $\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$

Let us simplify the expression,
$\sqrt{8} + \sqrt{50} + \sqrt{72} + \sqrt{98}$
$= \sqrt{(2\times4)} + \sqrt{(2\times25)} + \sqrt{(2\times36)} + \sqrt{(2\times49)}$
$= \sqrt{2}\sqrt{4} + \sqrt{2}\sqrt{25} + \sqrt{2}\sqrt{36} + \sqrt{2}\sqrt{49}$
$= 2\sqrt{2} + 5\sqrt{2} + 6\sqrt{2} + 7\sqrt{2}$
$= 20\sqrt{2}$
$= 20 \times 1.414$
$= 28.28$

(ii) $3\sqrt{32} - 2\sqrt{50} + 4\sqrt{128} - 20\sqrt{18}$
Let us simplify the expression,
$3\sqrt{32} - 2\sqrt{50} + 4\sqrt{128} - 20\sqrt{18}$
$= 3\sqrt{(16\times2)} - 2\sqrt{(25\times2)} + 4\sqrt{(64\times2)} - 20\sqrt{(9\times2)}$
$= 3\sqrt{16}\sqrt{2} - 2\sqrt{25}\sqrt{2} + 4\sqrt{64}\sqrt{2} - 20\sqrt{9}\sqrt{2}$
$= 3.4\sqrt{2} – 2.5\sqrt{2} + 4.8\sqrt{2} – 20.3\sqrt{2}$
$= 12\sqrt{2} - 10\sqrt{2} + 32\sqrt{2} - 60\sqrt{2}$
$= (12 – 10 + 32 - 60)\sqrt{2}$
$= -26\sqrt{2}$
$= -26 \times 1.414$
$= -36.764$

4. If $\sqrt{3} = 1.732$, then find the value of
(i) $\sqrt{27} + \sqrt{75} + \sqrt{108} - \sqrt{243}$
(ii) $5\sqrt{12} - 3\sqrt{48} + 6\sqrt{75} + 7\sqrt{108}$
Solution:
(i) $\sqrt{27} + \sqrt{75} + \sqrt{108} - \sqrt{243}$
Let us simplify the expression,
$\sqrt{27} + \sqrt{75} + \sqrt{108} - \sqrt{243}$
$= \sqrt{(9\times3)} + \sqrt{(25\times3)} + \sqrt{(36\times3)} - \sqrt{(81\times3)}$
$= \sqrt{9}\sqrt{3} + \sqrt{25}\sqrt{3} + \sqrt{36}\sqrt{3} - \sqrt{81}\sqrt{3}$
$= 3\sqrt{3} + 5\sqrt{3} + 6\sqrt{3} - 9\sqrt{3}$
$= (3 + 5 + 6 - 9)\sqrt{3}$
$= 5\sqrt{3}$
$= 5 \times 1.732$
$= 8.660$

(ii) $5\sqrt{12} - 3\sqrt{48} + 6\sqrt{75} + 7\sqrt{108}$
Let us simplify the expression,
$5\sqrt{12} - 3\sqrt{48} + 6\sqrt{75} + 7\sqrt{108}$

= 5√(4×3) - 3√(16×3) + 6√(25×3) + 7√(36×3)
= 5√4 √3 - 3√16 √3 + 6√25 √3 + 7√36 √3
= 5.2√3 – 3.4√3 + 6.5√3 + 7.6√3
= 10√3 - 12√3 + 30√3 + 42√3
= (10 – 12 + 30 + 42) √3
= 70√3
= 70 × 1.732
= 121.24

5. State which of the following are rational or irrational decimals.
(i) √(4/9), -3/70, √(7/25), √(16/5)
(ii) -√(2/49), 3/200, √(25/3), -√(49/16)
Solution:
(i) √(4/9), -3/70, √(7/25), √(16/5)
√(4/9) = 2/3
-3/70 = -3/70
√(7/25) = √7/5
√(16/5) = 4/√5
So,
√7/5 and 4/√5 are irrational decimals.
2/3 and -3/70 are rational decimals.

(ii) -√(2/49), 3/200, √(25/3), -√(49/16)
-√(2/49) = -√2/7
3/200 = 3/200
√(25/3) = 5/√3
-√(49/16) = -7/4
So,
-√2/7 and 5/√3 are irrational decimals.
3/200 and -7/4 are rational decimals.

6. State which of the following are rational or irrational decimals.
(i) -3√2
(ii) √(256/81)
(iii) √(27×16)
(iv) √(5/36)
Solution:
(i) -3√2
We know that √2 is an irrational number.

So, $-3\sqrt{2}$ will also be irrational number.

(ii) $\sqrt{(256/81)}$
$\sqrt{(256/81)} = 16/9 = 4/3$
It is a rational number.

(iii) $\sqrt{(27\times16)}$
$\sqrt{(27\times16)} = \sqrt{(9\times3\times16)} = 3\times4\sqrt{3} = 12\sqrt{3}$
It is an irrational number.

(iv) $\sqrt{(5/36)}$
$\sqrt{(5/36)} = \sqrt{5}/6$
It is an irrational number.

7. State which of the following are irrational numbers.
(i) $3 - \sqrt{(7/25)}$
(ii) $-2/3 + \sqrt[3]{2}$
(iii) $3/\sqrt{3}$
(iv) $-2/7 \sqrt[3]{5}$
(v) $(2 - \sqrt{3})(2 + \sqrt{3})$
(vi) $(3 + \sqrt{5})^2$
(vii) $(2/5 \sqrt{7})^2$
(viii) $(3 - \sqrt{6})^2$
Solution:
(i) $3 - \sqrt{(7/25)}$
Let us simplify,
$3 - \sqrt{(7/25)} = 3 - \sqrt{7}/\sqrt{25}$
$= 3 - \sqrt{7}/5$
Hence, $3 - \sqrt{7}/5$ is an irrational number.

(ii) $-2/3 + \sqrt[3]{2}$
Let us simplify,
$-2/3 + \sqrt[3]{2} = -2/3 + 2^{1/3}$
Since, 2 is not a perfect cube.
Hence it is an irrational number.

(iii) $3/\sqrt{3}$
Let us simplify,

By rationalizing, we get
$3/\sqrt{3} = 3\sqrt{3} /(\sqrt{3}\times\sqrt{3})$
$= 3\sqrt{3}/3$
$= \sqrt{3}$
Hence, $3/\sqrt{3}$ is an irrational number.

(iv) $-2/7 \sqrt[3]{5}$
Let us simplify,
$-2/7 \sqrt[3]{5} = -2/7 (5)^{1/3}$
Since, 5 is not a perfect cube.
Hence it is an irrational number.

(v) $(2 - \sqrt{3}) (2 + \sqrt{3})$
Let us simplify,
By using the formula,
$(a + b) (a - b) = (a)^2 \ (b)^2$
$(2 - \sqrt{3}) (2 + \sqrt{3}) = (2)^2 – (\sqrt{3})^2$
$= 4 – 3$
$= 1$
Hence, it is a rational number.

(vi) $(3 + \sqrt{5})^2$
Let us simplify,
By using $(a + b)^2 = a^2 + b^2 + 2ab$
$(3 + \sqrt{5})^2 = 3^2 + (\sqrt{5})^2 + 2.3.\sqrt{5}$
$= 9 + 5 + 6\sqrt{5}$
$= 14 + 6\sqrt{5}$
Hence, it is an irrational number.

(vii) $(2/5 \sqrt{7})^2$
Let us simplify,
$(2/5 \sqrt{7})^2 = (2/5 \sqrt{7}) \times (2/5 \sqrt{7})$
$= 4/ 25 \times 7$
$= 28/25$
Hence it is a rational number.

(viii) $(3 - \sqrt{6})^2$

Let us simplify,

By using $(a - b)^2 = a^2 + b^2 - 2ab$

$(3 - \sqrt{6})^2 = 3^2 + (\sqrt{6})^2 – 2.3.\sqrt{6}$

$= 9 + 6 - 6\sqrt{6}$

$= 15 - 6\sqrt{6}$

Hence it is an irrational number.

8. Prove the following are irrational numbers.

(i) $\sqrt[3]{2}$

(ii) $\sqrt[3]{3}$

(iii) $\sqrt[4]{5}$

Solution:

(i) $\sqrt[3]{2}$

We know that $\sqrt[3]{2} = 2^{1/3}$

Let us consider $2^{1/3} = p/q$, where p, q are integers, q>0.

p and q have no common factors (except 1).

So,

$2^{1/3} = p/q$

$2 = p^3/q^3$

$p^3 = 2q^3$ ….. (1)

We know that, 2 divides $2q^3$ then 2 divides p^3

So, 2 divides p

Now, let us consider p = 2k, where k is an integer

Substitute the value of p in (1), we get

$p^3 = 2q^3$

$(2k)^3 = 2q^3$

$8k^3 = 2q^3$

$4k^3 = q^3$

We know that, 2 divides $4k^3$ then 2 divides q^3

So, 2 divides q

Thus p and q have a common factor ‘2’.

This contradicts the statement, p and q have no common factor (except 1).

Hence, $\sqrt[3]{2}$ is an irrational number.

(ii) $\sqrt[3]{3}$

We know that $\sqrt[3]{3} = 3^{1/3}$

Let us consider $3^{1/3} = p/q$, where p, q are integers, q>0.

p and q have no common factors (except 1).

So,

$3^{1/3} = p/q$

$3 = p^3/q^3$

$p^3 = 3q^3$ ….. (1)

We know that, 3 divides $3q^3$ then 3 divides p^3

So, 3 divides p

Now, let us consider p = 3k, where k is an integer

Substitute the value of p in (1), we get

$p^3 = 3q^3$

$(3k)^3 = 3q^3$

$9k^3 = 3q^3$

$3k^3 = q^3$

We know that, 3 divides $9k^3$ then 3 divides q^3

So, 3 divides q

Thus p and q have a common factor '3'.

This contradicts the statement, p and q have no common factor (except 1).

Hence, $\sqrt[3]{3}$ is an irrational number.

(iii) $\sqrt[4]{5}$

We know that $\sqrt[4]{5} = 5^{1/4}$

Let us consider $5^{1/4} = p/q$, where p, q are integers, q>0.

p and q have no common factors (except 1).

So,

$5^{1/4} = p/q$

$5 = p^4/q^4$

$P^4 = 5q^4$ ….. (1)

We know that, 5 divides $5q^4$ then 5 divides p^4

So, 5 divides p

Now, let us consider p = 5k, where k is an integer

Substitute the value of p in (1), we get

$P^4 = 5q^4$

$(5k)^4 = 5q^4$

$625k^4 = 5q^4$

$125k^4 = q^4$

We know that, 5 divides $125k^4$ then 5 divides q^4

So, 5 divides q
Thus p and q have a common factor ‘5’.
This contradicts the statement, p and q have no common factor (except 1).
Hence, $\sqrt[4]{5}$ is an irrational number.

9. Find the greatest and the smallest real numbers.
(i) 2√3, 3/√2, -√7, √15
(ii) -3√2, 9/√5, -4, 4/3 √5, 3/2√3
Solution:
(i) 2√3, 3/√2, -√7, √15
Let us simplify each fraction
2√3 = √(4×3) = √12
3/√2 = (3×√2)/(√2×√2) = 3√2/2 = √((9/4)×2) = √(9/2) = √4.5
-√7 = -√7
√15 = √15
So,
The greatest real number = √15
Smallest real number = -√7

(ii) -3√2, 9/√5, -4, 4/3 √5, 3/2√3
Let us simplify each fraction
-3√2 = -√(9×2) = -√18
9/√5 = (9×√5)/(√5×√5) = 9√5/5 = √((81/25)×5) = √(81/5) = √16.2
-4 = -√16
4/3 √5 = √((16/9)×5) = √(80/9) = √8.88 = √8.8
3/2√3 = √((9/4)×3) = √(27/4) = √6.25
So,
The greatest real number = 9√5
Smallest real number = -3√2

10. Write in ascending order.
(i) 3√2, 2√3, √15, 4
(ii) 3√2, 2√8, 4, √50, 4√3
Solution:
(i) 3√2, 2√3, √15, 4
3√2 = √(9×2) =√18
2√3 = √(4×3) =√12
√15 = √15
4 = √16

Now, let us arrange in ascending order
$\sqrt{12}$, $\sqrt{15}$, $\sqrt{16}$, $\sqrt{18}$
So,
$2\sqrt{3}$, $\sqrt{15}$, 4, $3\sqrt{2}$

(ii) $3\sqrt{2}$, $2\sqrt{8}$, 4, $\sqrt{50}$, $4\sqrt{3}$
$3\sqrt{2} = \sqrt{(9\times2)} = \sqrt{18}$
$2\sqrt{8} = \sqrt{(4\times8)} = \sqrt{32}$
$4 = \sqrt{16}$
$\sqrt{50} = \sqrt{50}$
$4\sqrt{3} = \sqrt{(16\times3)} = \sqrt{48}$
Now, let us arrange in ascending order
$\sqrt{16}$, $\sqrt{18}$, $\sqrt{32}$, $\sqrt{48}$, $\sqrt{50}$
So,
4, $3\sqrt{2}$, $2\sqrt{8}$, $4\sqrt{3}$, $\sqrt{50}$

11. Write in descending order.
(i) $9/\sqrt{2}$, $3/2\ \sqrt{5}$, $4\sqrt{3}$, $3\sqrt{(6/5)}$
(ii) $5/\sqrt{3}$, $7/3\ \sqrt{2}$, $-\sqrt{3}$, $3\sqrt{5}$, $2\sqrt{7}$
Solution:
(i) $9/\sqrt{2}$, $3/2\ \sqrt{5}$, $4\sqrt{3}$, $3\sqrt{(6/5)}$
$9/\sqrt{2} = (9\times\sqrt{2})/(\sqrt{2}\times\sqrt{2}) = 9\sqrt{2}/2 = \sqrt{((81/4)\times2)} = \sqrt{(81/2)} = \sqrt{40.5}$
$3/2\ \sqrt{5} = \sqrt{((9/4)\times5)} = \sqrt{(45/4)} = \sqrt{11.25}$
$4\sqrt{3} = \sqrt{(16\times3)} = \sqrt{48}$
$3\sqrt{(6/5)} = \sqrt{((9\times6)/5)} = \sqrt{(54/5)} = \sqrt{10.8}$
Now, let us arrange in descending order
$\sqrt{48}$, $\sqrt{40.5}$, $\sqrt{11.25}$, $\sqrt{10.8}$
So,
$4\sqrt{3}$, $9/\sqrt{2}$, $3/2\ \sqrt{5}$, $3\sqrt{(6/5)}$

(ii) $5/\sqrt{3}$, $7/3\ \sqrt{2}$, $-\sqrt{3}$, $3\sqrt{5}$, $2\sqrt{7}$
$5/\sqrt{3} = \sqrt{(25/3)} = \sqrt{8.33}$
$7/3\ \sqrt{2} = \sqrt{((49/9)\times2)} = \sqrt{98/9} = \sqrt{10.88}$
$-\sqrt{3} = -\sqrt{3}$
$3\sqrt{5} = \sqrt{(9\times5)} = \sqrt{45}$
$2\sqrt{7} = \sqrt{(4\times7)} = \sqrt{28}$
Now, let us arrange in descending order
$\sqrt{45}$, $\sqrt{28}$, $\sqrt{10.88..}$, $\sqrt{8.33..}$, $-\sqrt{3}$
So,

$3\sqrt{5}, 2\sqrt{7}, 7/3\sqrt{2}, 5/\sqrt{3}, -\sqrt{3}$

12. Arrange in ascending order.
$\sqrt[3]{2}, \sqrt{3}, \sqrt[6]{5}$
Solution:
Here we can express the given expressions as:
$\sqrt[3]{2} = 2^{1/3}$
$\sqrt{3} = 3^{1/2}$
$\sqrt[6]{5} = 5^{1/6}$
Let us make the roots common so,
$2^{1/3} = 2^{(2 \times 1/2 \times 1/3)} = 4^{1/6}$
$3^{1/2} = 3^{(3 \times 1/3 \times 1/2)} = 27^{1/6}$
$5^{1/6} = 5^{1/6}$
Now, let us arrange in ascending order,
$4^{1/6}, 5^{1/6}, 27^{1/6}$
So,
$2^{1/3}, 5^{1/6}, 3^{1/2}$
So,
$\sqrt[3]{2}, \sqrt[6]{5}, \sqrt{3}$

EXERCISE 1.5

1. Rationalize the following:
(i) $3/4\sqrt{5}$
(ii) $5\sqrt{7} / \sqrt{3}$
(iii) $3/(4 - \sqrt{7})$
(iv) $17/(3\sqrt{2} + 1)$
(v) $16/ (\sqrt{41} - 5)$
(vi) $1/ (\sqrt{7} - \sqrt{6})$
(vii) $1/ (\sqrt{5} + \sqrt{2})$
(viii) $(\sqrt{2} + \sqrt{3}) / (\sqrt{2} - \sqrt{3})$
Solution:
(i) $3/4\sqrt{5}$
Let us rationalize,
$3/4\sqrt{5} = (3\times\sqrt{5}) /(4\sqrt{5}\times\sqrt{5})$
$= (3\sqrt{5}) / (4\times5)$
$= (3\sqrt{5}) / 20$

(ii) $5\sqrt{7} / \sqrt{3}$
Let us rationalize,
$5\sqrt{7} / \sqrt{3} = (5\sqrt{7}\times\sqrt{3}) / (\sqrt{3}\times\sqrt{3})$
$= 5\sqrt{21}/3$

(iii) $3/(4 - \sqrt{7})$
Let us rationalize,
$3/(4 - \sqrt{7}) = [3\times(4 + \sqrt{7})] / [(4 - \sqrt{7}) \times (4 + \sqrt{7})]$
$= 3(4 + \sqrt{7}) / [4^2 - (\sqrt{7})^2]$
$= 3(4 + \sqrt{7}) / [16 - 7]$
$= 3(4 + \sqrt{7}) / 9$
$= (4 + \sqrt{7}) / 3$

(iv) $17/(3\sqrt{2} + 1)$
Let us rationalize,
$17/(3\sqrt{2} + 1) = 17(3\sqrt{2} - 1) / [(3\sqrt{2} + 1) (3\sqrt{2} - 1)]$
$= 17(3\sqrt{2} - 1) / [(3\sqrt{2})^2 - 1^2]$
$= 17(3\sqrt{2} - 1) / [9.2 - 1]$
$= 17(3\sqrt{2} - 1) / [18 - 1]$
$= 17(3\sqrt{2} - 1) / 17$
$= (3\sqrt{2} - 1)$

(v) $16/(\sqrt{41} - 5)$

Let us rationalize,

$16/(\sqrt{41} - 5) = 16(\sqrt{41} + 5) / [(\sqrt{41} - 5)(\sqrt{41} + 5)]$

$= 16(\sqrt{41} + 5) / [(\sqrt{41})^2 - 5^2]$

$= 16(\sqrt{41} + 5) / [41 - 25]$

$= 16(\sqrt{41} + 5) / [16]$

$= (\sqrt{41} + 5)$

(vi) $1/(\sqrt{7} - \sqrt{6})$

Let us rationalize,

$1/(\sqrt{7} - \sqrt{6}) = 1(\sqrt{7} + \sqrt{6}) / [(\sqrt{7} - \sqrt{6})(\sqrt{7} + \sqrt{6})]$

$= (\sqrt{7} + \sqrt{6}) / [(\sqrt{7})^2 – (\sqrt{6})^2]$

$= (\sqrt{7} + \sqrt{6}) / [7 - 6]$

$= (\sqrt{7} + \sqrt{6}) / 1$

$= (\sqrt{7} + \sqrt{6})$

(vii) $1/(\sqrt{5} + \sqrt{2})$

Let us rationalize,

$1/(\sqrt{5} + \sqrt{2}) = 1(\sqrt{5} - \sqrt{2}) / [(\sqrt{5} + \sqrt{2})(\sqrt{5} - \sqrt{2})]$

$= (\sqrt{5} - \sqrt{2}) / [(\sqrt{5})^2 – (\sqrt{2})^2]$

$= (\sqrt{5} - \sqrt{2}) / [5 - 2]$

$= (\sqrt{5} - \sqrt{2}) / [3]$

$= (\sqrt{5} - \sqrt{2}) /3$

(viii) $(\sqrt{2} + \sqrt{3}) / (\sqrt{2} - \sqrt{3})$

Let us rationalize,

$(\sqrt{2} + \sqrt{3}) / (\sqrt{2} - \sqrt{3}) = [(\sqrt{2} + \sqrt{3})(\sqrt{2} + \sqrt{3})] / [(\sqrt{2} - \sqrt{3})(\sqrt{2} + \sqrt{3})]$

$= [(\sqrt{2} + \sqrt{3})^2] / [(\sqrt{2})^2 – (\sqrt{3})^2]$

$= [2 + 3 + 2\sqrt{2}\sqrt{3}] / [2 - 3]$

$= [5 + 2\sqrt{6}] / -1$

$= - (5 + 2\sqrt{6})$

2. Simplify:

(i) $(7 + 3\sqrt{5}) / (7 - 3\sqrt{5})$

(ii) $(3 - 2\sqrt{2}) / (3 + 2\sqrt{2})$

(iii) $(5 - 3\sqrt{14}) / (7 + 2\sqrt{14})$

Solution:

(i) $(7 + 3\sqrt{5}) / (7 - 3\sqrt{5})$

Let us rationalize the denominator, we get

$(7 + 3\sqrt{5}) / (7 - 3\sqrt{5}) = [(7 + 3\sqrt{5}) (7 + 3\sqrt{5})] / [(7 - 3\sqrt{5}) (7 + 3\sqrt{5})]$

$= [(7 + 3\sqrt{5})^2] / [7^2 – (3\sqrt{5})^2]$

$= [7^2 + (3\sqrt{5})^2 + 2.7. 3\sqrt{5}] / [49 – 9.5]$

$= [49 + 9.5 + 42\sqrt{5}] / [49 - 45]$

$= [49 + 45 + 42\sqrt{5}] / [4]$

$= [94 + 42\sqrt{5}] / 4$

$= 2[47 + 21\sqrt{5}]/4$

$= [47 + 21\sqrt{5}]/2$

(ii) $(3 - 2\sqrt{2}) / (3 + 2\sqrt{2})$

Let us rationalize the denominator, we get

$(3 - 2\sqrt{2}) / (3 + 2\sqrt{2}) = [(3 - 2\sqrt{2}) (3 - 2\sqrt{2})] / [(3 + 2\sqrt{2}) (3 - 2\sqrt{2})]$

$= [(3 - 2\sqrt{2})^2] / [3^2 – (2\sqrt{2})^2]$

$= [3^2 + (2\sqrt{2})^2 – 2.3.2\sqrt{2}] / [9 – 4.2]$

$= [9 + 4.2 - 12\sqrt{2}] / [9 - 8]$

$= [9 + 8 - 12\sqrt{2}] / 1$

$= 17 - 12\sqrt{2}$

(iii) $(5 - 3\sqrt{14}) / (7 + 2\sqrt{14})$

Let us rationalize the denominator, we get

$(5 - 3\sqrt{14}) / (7 + 2\sqrt{14}) = [(5 - 3\sqrt{14}) (7 - 2\sqrt{14})] / [(7 + 2\sqrt{14}) (7 - 2\sqrt{14})]$

$= [5(7 - 2\sqrt{14}) – 3\sqrt{14} (7 - 2\sqrt{14})] / [7^2 – (2\sqrt{14})^2]$

$= [35 - 10\sqrt{14} - 21\sqrt{14} + 6.14] / [49 – 4.14]$

$= [35 - 31\sqrt{14} + 84] / [49 - 56]$

$= [119 - 31\sqrt{14}] / [-7]$

$= -[119 - 31\sqrt{14}] / 7$

$= [31\sqrt{14} – 119] / 7$

3. Simplify:

$[7\sqrt{3} / (\sqrt{10} + \sqrt{3})] – [2\sqrt{5} / (\sqrt{6} + \sqrt{5})] – [3\sqrt{2} / (\sqrt{15} + 3\sqrt{2})]$

Solution:

Let us simplify individually,

$[7\sqrt{3} / (\sqrt{10} + \sqrt{3})]$

Let us rationalize the denominator,

$7\sqrt{3} / (\sqrt{10} + \sqrt{3}) = [7\sqrt{3}(\sqrt{10} - \sqrt{3})] / [(\sqrt{10} + \sqrt{3}) (\sqrt{10} - \sqrt{3})]$

$= [7\sqrt{3}.\sqrt{10} - 7\sqrt{3}.\sqrt{3}] / [(\sqrt{10})^2 – (\sqrt{3})^2]$

$= [7\sqrt{30} – 7.3] / [10 - 3]$

$= 7[\sqrt{30} - 3] / 7$

$= \sqrt{30} - 3$

Now,

$[2\sqrt{5} / (\sqrt{6} + \sqrt{5})]$

Let us rationalize the denominator, we get

$2\sqrt{5} / (\sqrt{6} + \sqrt{5}) = [2\sqrt{5} (\sqrt{6} - \sqrt{5})] / [(\sqrt{6} + \sqrt{5}) (\sqrt{6} - \sqrt{5})]$

$= [2\sqrt{5}.\sqrt{6} - 2\sqrt{5}.\sqrt{5}] / [(\sqrt{6})^2 – (\sqrt{5})^2]$

$= [2\sqrt{30} – 2.5] / [6 - 5]$

$= [2\sqrt{30} - 10] / 1$

$= 2\sqrt{30} – 10$

Now,

$[3\sqrt{2} / (\sqrt{15} + 3\sqrt{2})]$

Let us rationalize the denominator, we get

$3\sqrt{2} / (\sqrt{15} + 3\sqrt{2}) = [3\sqrt{2} (\sqrt{15} - 3\sqrt{2})] / [(\sqrt{15} + 3\sqrt{2}) (\sqrt{15} - 3\sqrt{2})]$

$= [3\sqrt{2}.\sqrt{15} - 3\sqrt{2}.3\sqrt{2}] / [(\sqrt{15})^2 – (3\sqrt{2})^2]$

$= [3\sqrt{30} – 9.2] / [15 – 9.2]$

$= [3\sqrt{30} - 18] / [15 - 18]$

$= 3[\sqrt{30} - 6] / [-3]$

$= [\sqrt{30} - 6] / -1$

$= 6 - \sqrt{30}$

So, according to the question let us substitute the obtained values,

$[7\sqrt{3} / (\sqrt{10} + \sqrt{3})] – [2\sqrt{5} / (\sqrt{6} + \sqrt{5})] – [3\sqrt{2} / (\sqrt{15} + 3\sqrt{2})]$

$= (\sqrt{30} – 3) – (2\sqrt{30} – 10) – (6 - \sqrt{30})$

$= \sqrt{30} – 3 - 2\sqrt{30} + 10 – 6 + \sqrt{30}$

$= 2\sqrt{30} - 2\sqrt{30} – 3 + 10 – 6$

$= 1$

4. Simplify:

$[1/(\sqrt{4} + \sqrt{5})] + [1/(\sqrt{5} + \sqrt{6})] + [1/(\sqrt{6} + \sqrt{7})] + [1/(\sqrt{7} + \sqrt{8})] + [1/(\sqrt{8} + \sqrt{9})]$

Solution:

Let us simplify individually,

$[1/(\sqrt{4} + \sqrt{5})]$

Rationalize the denominator, we get

$[1/(\sqrt{4} + \sqrt{5})] = [1(\sqrt{4} - \sqrt{5})] / [(\sqrt{4} + \sqrt{5}) (\sqrt{4} - \sqrt{5})]$

$= [(\sqrt{4} - \sqrt{5})] / [(\sqrt{4})^2 – (\sqrt{5})^2]$

$= [(\sqrt{4} - \sqrt{5})] / [4 - 5]$

$= [(\sqrt{4} - \sqrt{5})] / -1$

$= -(\sqrt{4} - \sqrt{5})$

Now,

$[1/(\sqrt{5} + \sqrt{6})]$

Rationalize the denominator, we get

$[1/(\sqrt{5} + \sqrt{6})] = [1(\sqrt{5} - \sqrt{6})] / [(\sqrt{5} + \sqrt{6})(\sqrt{5} - \sqrt{6})]$

$= [(\sqrt{5} - \sqrt{6})] / [(\sqrt{5})^2 – (\sqrt{6})^2]$

$= [(\sqrt{5} - \sqrt{6})] / [5 - 6]$

$= [(\sqrt{5} - \sqrt{6})] / -1$

$= -(\sqrt{5} - \sqrt{6})$

Now,

$[1/(\sqrt{6} + \sqrt{7})]$

Rationalize the denominator, we get

$[1/(\sqrt{6} + \sqrt{7})] = [1(\sqrt{6} - \sqrt{7})] / [(\sqrt{6} + \sqrt{7})(\sqrt{6} - \sqrt{7})]$

$= [(\sqrt{6} - \sqrt{7})] / [(\sqrt{6})^2 – (\sqrt{7})^2]$

$= [(\sqrt{6} - \sqrt{7})] / [6 - 7]$

$= [(\sqrt{6} - \sqrt{7})] / -1$

$= -(\sqrt{6} - \sqrt{7})$

Now,

$[1/(\sqrt{7} + \sqrt{8})]$

Rationalize the denominator, we get

$[1/(\sqrt{7} + \sqrt{8})] = [1(\sqrt{7} - \sqrt{8})] / [(\sqrt{7} + \sqrt{8})(\sqrt{7} - \sqrt{8})]$

$= [(\sqrt{7} - \sqrt{8})] / [(\sqrt{7})^2 – (\sqrt{8})^2]$

$= [(\sqrt{7} - \sqrt{8})] / [7 - 8]$

$= [(\sqrt{7} - \sqrt{8})] / -1$

$= -(\sqrt{7} - \sqrt{8})$

Now,

$[1/(\sqrt{8} + \sqrt{9})]$

Rationalize the denominator, we get

$[1/(\sqrt{8} + \sqrt{9})] = [1(\sqrt{8} - \sqrt{9})] / [(\sqrt{8} + \sqrt{9})(\sqrt{8} - \sqrt{9})]$

$= [(\sqrt{8} - \sqrt{9})] / [(\sqrt{8})^2 – (\sqrt{9})^2]$

$= [(\sqrt{8} - \sqrt{9})] / [8 - 9]$

$= [(\sqrt{8} - \sqrt{9})] / -1$

$= -(\sqrt{8} - \sqrt{9})$

So, according to the question let us substitute the obtained values,

$[1/(\sqrt{4} + \sqrt{5})] + [1/(\sqrt{5} + \sqrt{6})] + [1/(\sqrt{6} + \sqrt{7})] + [1/(\sqrt{7} + \sqrt{8})] + [1/(\sqrt{8} + \sqrt{9})]$

$= -(\sqrt{4} - \sqrt{5}) + -(\sqrt{5} - \sqrt{6}) + -(\sqrt{6} - \sqrt{7}) + -(\sqrt{7} - \sqrt{8}) + -(\sqrt{8} - \sqrt{9})$

$= -\sqrt{4} + \sqrt{5} - \sqrt{5} + \sqrt{6} - \sqrt{6} + \sqrt{7} - \sqrt{7} + \sqrt{8} - \sqrt{8} + \sqrt{9}$

$= -\sqrt{4} + \sqrt{9}$

$= -2 + 3$

$= 1$

5. Given, find the value of a and b, if
(i) $[3 - \sqrt{5}] / [3 + 2\sqrt{5}] = -19/11 + a\sqrt{5}$
(ii) $[\sqrt{2} + \sqrt{3}] / [3\sqrt{2} - 2\sqrt{3}] = a - b\sqrt{6}$
(iii) $\{[7 + \sqrt{5}]/[7 - \sqrt{5}]\} – \{[7 - \sqrt{5}]/[7 + \sqrt{5}]\} = a + 7/11\ b\sqrt{5}$
Solution:
(i) $[3 - \sqrt{5}] / [3 + 2\sqrt{5}] = -19/11 + a\sqrt{5}$
Let us consider LHS
$[3 - \sqrt{5}] / [3 + 2\sqrt{5}]$
Rationalize the denominator,
$[3 - \sqrt{5}] / [3 + 2\sqrt{5}] = [(3 - \sqrt{5}) (3 - 2\sqrt{5})] / [(3 + 2\sqrt{5}) (3 - 2\sqrt{5})]$
$= [3(3 - 2\sqrt{5}) – \sqrt{5}(3 - 2\sqrt{5})] / [3^2 – (2\sqrt{5})^2]$
$= [9 - 6\sqrt{5} - 3\sqrt{5} + 2.5] / [9 – 4.5]$
$= [9 - 6\sqrt{5} - 3\sqrt{5} + 10] / [9 – 20]$
$= [19 - 9\sqrt{5}] / -11$
$= -19/11 + 9\sqrt{5}/11$
So when comparing with RHS
$-19/11 + 9\sqrt{5}/11 = -19/11 + a\sqrt{5}$
Hence, value of a = 9/11

(ii) $[\sqrt{2} + \sqrt{3}] / [3\sqrt{2} - 2\sqrt{3}] = a - b\sqrt{6}$
Let us consider LHS
$[\sqrt{2} + \sqrt{3}] / [3\sqrt{2} - 2\sqrt{3}]$
Rationalize the denominator,
$[\sqrt{2} + \sqrt{3}] / [3\sqrt{2} - 2\sqrt{3}] = [(\sqrt{2} + \sqrt{3}) (3\sqrt{2} + 2\sqrt{3})] / [(3\sqrt{2} - 2\sqrt{3}) (3\sqrt{2} + 2\sqrt{3})]$
$= [\sqrt{2}(3\sqrt{2} + 2\sqrt{3}) + \sqrt{3}(3\sqrt{2} + 2\sqrt{3})] / [(3\sqrt{2})^2 – (2\sqrt{3})^2]$
$= [3.2 + 2\sqrt{2}\sqrt{3} + 3\sqrt{2}\sqrt{3} + 2.3] / [9.2 – 4.3]$
$= [6 + 2\sqrt{6} + 3\sqrt{6} + 6] / [18 - 12]$
$= [12 + 5\sqrt{6}] / 6$
$= 12/6 + 5\sqrt{6}/6$
$= 2 + 5\sqrt{6}/6$
$= 2 – (-5\sqrt{6}/6)$
So when comparing with RHS
$2 – (-5\sqrt{6}/6) = a - b\sqrt{6}$
Hence, value of a = 2 and b = -5/6

(iii) $\{[7 + \sqrt{5}]/[7 - \sqrt{5}]\} – \{[7 - \sqrt{5}]/[7 + \sqrt{5}]\} = a + 7/11\ b\sqrt{5}$
Let us consider LHS
Since there are two terms, let us solve individually
$\{[7 + \sqrt{5}]/[7 - \sqrt{5}]\}$

Rationalize the denominator,

$[7 + \sqrt{5}]/[7 - \sqrt{5}] = [(7 + \sqrt{5})(7 + \sqrt{5})] / [(7 - \sqrt{5})(7 + \sqrt{5})]$

$= [(7 + \sqrt{5})^2] / [7^2 – (\sqrt{5})^2]$

$= [7^2 + (\sqrt{5})^2 + 2.7.\sqrt{5}] / [49 - 5]$

$= [49 + 5 + 14\sqrt{5}] / [44]$

$= [54 + 14\sqrt{5}] / 44$

Now,

$\{[7 - \sqrt{5}]/[7 + \sqrt{5}]\}$

Rationalize the denominator,

$[7 - \sqrt{5}]/[7 + \sqrt{5}] = (7 - \sqrt{5})(7 - \sqrt{5})] / [(7 + \sqrt{5})(7 - \sqrt{5})]$

$= [(7 - \sqrt{5})^2] / [7^2 – (\sqrt{5})^2]$

$= [7^2 + (\sqrt{5})^2 - 2.7.\sqrt{5}] / [49 - 5]$

$= [49 + 5 - 14\sqrt{5}] / [44]$

$= [54 - 14\sqrt{5}] / 44$

So, according to the question

$\{[7 + \sqrt{5}]/[7 - \sqrt{5}]\} – \{[7 - \sqrt{5}]/[7 + \sqrt{5}]\}$

By substituting the obtained values,

$= \{[54 + 14\sqrt{5}] / 44\} – \{[54 - 14\sqrt{5}] / 44\}$

$= [54 + 14\sqrt{5} – 54 + 14\sqrt{5}]/44$

$= 28\sqrt{5}/44$

$= 7\sqrt{5}/11$

So when comparing with RHS

$7\sqrt{5}/11 = a + 7/11\ b\sqrt{5}$

Hence, value of a = 0 and b = 1

6. Simplify:

$\{[7 + 3\sqrt{5}] / [3 + \sqrt{5}]\} – \{[7 - 3\sqrt{5}] / [3 - \sqrt{5}]\} = p + q\sqrt{5}$

Solution:

Let us consider LHS

Since there are two terms, let us solve individually

$\{[7 + 3\sqrt{5}] / [3 + \sqrt{5}]\}$

Rationalize the denominator,

$[7 + 3\sqrt{5}] / [3 + \sqrt{5}] = [(7 + 3\sqrt{5})(3 - \sqrt{5})] / [(3 + \sqrt{5})(3 - \sqrt{5})]$

$= [7(3 - \sqrt{5}) + 3\sqrt{5}(3 - \sqrt{5})] / [3^2 – (\sqrt{5})^2]$

$= [21 - 7\sqrt{5} + 9\sqrt{5} – 3.5] / [9 - 5]$

$= [21 + 2\sqrt{5} - 15] / [4]$

$= [6 + 2\sqrt{5}] / 4$

$= 2[3 + \sqrt{5}]/4$

$= [3 + \sqrt{5}] /2$

Now,

$\{[7 - 3\sqrt{5}] / [3 - \sqrt{5}]\}$

Rationalize the denominator,

$[7 - 3\sqrt{5}] / [3 - \sqrt{5}] = [(7 - 3\sqrt{5})(3 + \sqrt{5})] / [(3 - \sqrt{5})(3 + \sqrt{5})]$
$= [7(3 + \sqrt{5}) – 3\sqrt{5}(3 + \sqrt{5})] / [3^2 – (\sqrt{5})^2]$
$= [21 + 7\sqrt{5} - 9\sqrt{5} – 3.5] / [9 - 5]$
$= [21 - 2\sqrt{5} - 15] / 4$
$= [6 - 2\sqrt{5}]/4$
$= 2[3 - \sqrt{5}]/4$
$= [3 - \sqrt{5}]/2$

So, according to the question

$\{[7 + 3\sqrt{5}] / [3 + \sqrt{5}]\} – \{[7 - 3\sqrt{5}] / [3 - \sqrt{5}]\}$

By substituting the obtained values,

$= \{[3 + \sqrt{5}] /2\} – \{[3 - \sqrt{5}] /2\}$
$= [3 + \sqrt{5} – 3 + \sqrt{5}]/2$
$= [2\sqrt{5}]/2$
$= \sqrt{5}$

So when comparing with RHS

$\sqrt{5} = p + q\sqrt{5}$

Hence, value of p = 0 and q = 1

7. If $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, find
(i) $\sqrt{2}/(2 + \sqrt{2})$
(ii) $1/(\sqrt{3} + \sqrt{2})$
Solution:

(i) $\sqrt{2}/(2 + \sqrt{2})$

By rationalizing the denominator,

$\sqrt{2}/(2 + \sqrt{2}) = [\sqrt{2}(2 - \sqrt{2})] / [(2 + \sqrt{2})(2 - \sqrt{2})]$
$= [2\sqrt{2} – 2] / [2^2 – (\sqrt{2})^2]$
$= [2\sqrt{2} - 2] / [4 - 2]$
$= 2[\sqrt{2} - 1] / 2$
$= \sqrt{2} – 1$
$= 1.414 – 1$
$= 0.414$

(ii) $1/(\sqrt{3} + \sqrt{2})$

By rationalizing the denominator,

$1/(\sqrt{3} + \sqrt{2}) = [1(\sqrt{3} - \sqrt{2})] / [(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})]$
$= [(\sqrt{3} - \sqrt{2})] / [(\sqrt{3})^2 – (\sqrt{2})^2]$

$= [(\sqrt{3} - \sqrt{2})] / [3 - 2]$
$= [(\sqrt{3} - \sqrt{2})] / 1$
$= (\sqrt{3} - \sqrt{2})$
$= 1.732 – 1.414$
$= 0.318$

8. If a = 2 + √3, find 1/a, (a – 1/a)
Solution:
Given:
$a = 2 + \sqrt{3}$
So,
$1/a = 1/ (2 + \sqrt{3})$
By rationalizing the denominator,
$1/ (2 + \sqrt{3}) = [1(2 - \sqrt{3})] / [(2 + \sqrt{3}) (2 - \sqrt{3})]$
$= [(2 - \sqrt{3})] / [2^2 – (\sqrt{3})^2]$
$= [(2 - \sqrt{3})] / [4 - 3]$
$= (2 - \sqrt{3})$
Then,
$a – 1/a = 2 + \sqrt{3} – (2 - \sqrt{3})$
$= 2 + \sqrt{3} – 2 + \sqrt{3}$
$= 2\sqrt{3}$

9. Solve:
If x = 1 - √2, find 1/x, (x – 1/x)^4
Solution:
Given:
$x = 1 - \sqrt{2}$
so,
$1/x = 1/(1 - \sqrt{2})$
By rationalizing the denominator,
$1/ (1 - \sqrt{2}) = [1(1 + \sqrt{2})] / [(1 - \sqrt{2}) (1 + \sqrt{2})]$
$= [(1 + \sqrt{2})] / [1^2 – (\sqrt{2})^2]$
$= [(1 + \sqrt{2})] / [1 - 2]$
$= (1 + \sqrt{2}) / -1$
$= -(1 + \sqrt{2})$
Then,
$(x – 1/x)^4 = [1 - \sqrt{2} – (-1 - \sqrt{2})]^4$
$= [1 - \sqrt{2} + 1 + \sqrt{2}]^4$
$= 2^4$

= 16

10. Solve:
If x = 5 - 2√6, find 1/x, (x^2 – 1/x^2)
Solution:
Given:
x = 5 – 2√6
so,
1/x = 1/(5 - 2√6)
By rationalizing the denominator,
1/(5 - 2√6) = [1(5 + 2√6)] / [(5 - 2√6) (5 + 2√6)]
= [(5 + 2√6)] / [5^2 – $(2\sqrt{6})^2$]
= [(5 + 2√6)] / [25 – 4.6]
= [(5 + 2√6)] / [25 - 24]
= (5 + 2√6)
Then,
x + 1/x = 5 – 2√6 + (5 + 2√6)
= 10
Square on both sides we get
$(x + 1/x)^2 = 10^2$
$x^2 + 1/x^2 + 2x.1/x = 100$
$x^2 + 1/x^2 + 2 = 100$
$x^2 + 1/x^2 = 100 – 2$
= 98

11. If p = (2-√5)/(2+√5) and q = (2+√5)/(2-√5), find the values of
(i) p + q
(ii) p – q
(iii) $p^2 + q^2$
(iv) $p^2 – q^2$
Solution:
Given:
p = (2-√5)/(2+√5) and q = (2+√5)/(2-√5)
(i) p + q
[(2-√5)/(2+√5)] + [(2+√5)/(2-√5)]
So by rationalizing the denominator, we get
= [$(2 - \sqrt{5})^2 + (2 + \sqrt{5})^2$] / [$2^2 – (\sqrt{5})^2$]
= [4 + 5 - 4√5 + 4 + 5 + 4√5] / [4 - 5]
= [18]/-1

= -18

(ii) p – q
[(2-√5)/(2+√5)] - [(2+√5)/(2-√5)]
So by rationalizing the denominator, we get
$= [(2 - \sqrt{5})^2 - (2 + \sqrt{5})^2] / [2^2 – (\sqrt{5})^2]$
= [4 + 5 - 4√5 – (4 + 5 + 4√5)] / [4 - 5]
= [9 - 4√5 – 9 - 4√5] / -1
= [-8√5]/-1
= 8√5

(iii) $p^2 + q^2$
We know that $(p + q)^2 = p^2 + q^2 + 2pq$
So,
$p^2 + q^2 = (p + q)^2 – 2pq$
pq = [(2-√5)/(2+√5)] × [(2+√5)/(2-√5)]
= 1
p + q = -18
so,
$p^2 + q^2 = (p + q)^2 – 2pq$
$= (-18)^2 – 2(1)$
= 324 – 2
= 322

(iv) $p^2 – q^2$
We know that, $p^2 – q^2 = (p + q) (p - q)$
So, by substituting the values
$p^2 – q^2 = (p + q) (p - q)$
= (-18) (8√5)
= -144√5

12. If x = (√2 - 1)/(√2 + 1) and y = (√2 + 1)/(√2 - 1), find
(i) x + y
(ii) xy
Solution:
Given:
x = (√2 - 1)/(√2 + 1) and y = (√2 + 1)/(√2 - 1)
(i) x + y
= [(√2 - 1)/(√2 + 1)] + [(√2 + 1)/(√2 - 1)]

By rationalizing the denominator,

$= [(\sqrt{2} - 1)^2 + (\sqrt{2} + 1)^2] / [(\sqrt{2})^2 - 1^2]$

$= [2 + 1 - 2\sqrt{2} + 2 + 1 + 2\sqrt{2}] / [2 - 1]$

$= [6] / 1$

$= 6$

(ii) xy

$[(\sqrt{2} - 1)/(\sqrt{2} + 1)] \times [(\sqrt{2} + 1)/(\sqrt{2} - 1)]$

$= 1$